Solving Systems of Equations Worksheet

Solve the following systems for the unknown variables. Some might have no solution or have infinite solutions.

1. $x - 2y = -6$
   $y - 3x = 3$
2. $5m = n - 10$
   $5m = n + 10$
3. $-2h = -3k + 5$
   $-4h = -6k + 10$

Economics

In economics, a market is said to be in equilibrium when quantity demanded (denoted Q_D) is equal to quantity supplied (denoted Q_S). Also suppose that quantity is in thousands of units (so $Q = 3.2$ means a quantity of 3200 goods). The price at which the market is equilibrium is called the equilibrium or market clearing price. Use this fact and your knowledge of systems of equations to solve question 4 and 5.

4. Suppose that demand equation is $P = 9 - 4Q_D$ and the supply equation is $Q_S = 11 - 9P$. What is the equilibrium price and quantity?

5. Suppose that demand equation is $P = 9 - 4Q_D$ and the supply equation is $Q_S = 11 - 9P$. Suppose the market price is \$3. Is the market in equilibrium?

6. A certain amount of money was deposited at an 8% annual interest rate and a second amount was deposited at a 5% annual interest rate. The total amount of money deposited for one year was \$1500 and the interest rate accumulated from the two investments was \$99. Find how much was deposited at each rate.

Education

7. A student has two test scores. The difference between the two scores is 12 and the mean of the scores is 80. What are the two test scores?

8. For every exam taken, a student receives 3 points for a correct answer and has two points deducted for every wrong answer. If there are 40 questions on the test and the student received a grade of 85, how many questions did she get correct and how many were incorrect?

9. For purposes of comparison, a teacher wants to rescale the scores on a difficult set of papers so that the maximum possible is still 100 but the mean (average) is 80 instead of 56. Write a linear formula which would do this and then decide if 60 is the lowest passing mark, what was the lowest passing mark on the original scale?

General

10. A school is going on a fieldtrip. For every 8 students present there has to be one chaperone. There are 81 people on the fieldtrip. How many are chaperones and how many are students?

Chemistry

11. Chemical equations can be balanced as in the following example. Beginning with the unbalanced equation

$$Ca + H_3PO \rightarrow Ca_3P_2O_8 + H_2.$$

The problem is to determine numbers of molecules of each of the four chemicals so that the equation will be balanced. We want

$$(\mathrm{w})Ca + (x)H_3PO \rightarrow (y)Ca_3P_2O_8 + (z)H_2,$$

where w, x ,y, z are numbers of molecules of the respective compounds. Equating the number of atoms of each elements gives the following: Calcium: $w = 3y$, Hydrogen: $3x = 2z$, Phosphorus: $x = 2y$, Oxygen: $4x = 8y$. Find the smallest solution in integers.

Systems of Equations

① $x-2y=-6$

$y-3x=3$

First, solve for y → $y=3+3x$ → plug y into 1st equation

$$x-2(3+3x)=-6$$

$$x-\cancel{6}-6x=-6$$

$$+\cancel{6} \qquad +6$$

$$x-6x=0$$

$$-5x=0$$

$$\frac{-5x}{-5}=\frac{0}{-5}$$

$\boxed{x=0}$ → plug back into 2nd eqtn to find y

$$y-3(0)=3$$

$$y-0=3$$

$$\boxed{y=3}$$

Check:

$x-2y=-6$

$0-2(3)=-6$

$0-6=-6$

$-6=-6$ ✓

$y-3x=3$

$3-3(0)=3$

$3-0=3$

$3=3$ ✓

② $5m=n-10$

$5m=n+10$

Solve $5m=n-10$ for n

$n=5m+10$

Sub above into $5m=n+10$

$5m=(5m+10)+10$

$5m=5m+10+10$

$5m=5m+20$

$-5m \quad -5m$

$0=20$ ✗

Solve $5m=n+10$ for n

$5m=n+10$

$-10 \quad -10$

$5m-10=n$

Plug into 1st equation

$5m=n-10$

$5m=(5m-10)-10$

$5m=5m-10-10$

$5m=5m-20$

$-5m \quad -5m$

$0=-20$ ✗

$\boxed{\text{no solution!}}$

③ $-2h = -3k + 5$

$-4h = -6k + 10$

Solve $\frac{-4h}{-4} = \frac{-6k+10}{-4}$ for h

$h = -\frac{6k+10}{-4}$

$h = \frac{6}{4}k + \frac{10}{-4}$

$h = \frac{3}{2}k - \frac{5}{2}$

Plug h into 1st eqtn:

$-2h = -3k + 5$

$-2\left(\frac{3}{2}k - \frac{5}{2}\right) = -3k + 5$

$-\frac{6}{2}k + \frac{10}{2} = -3k + 5$

$-\cancel{3k} + 5 = -\cancel{3}k + 5$

$+3k \qquad +3k$

$5 = 5$ ✓

Solve $\frac{-2h}{-2} = \frac{-3k+5}{-2}$ for h

$h = \frac{3}{2}k - \frac{5}{2}$

Plug into 2nd eqtn:

$-4\left(\frac{3}{2}k - \frac{5}{2}\right) = -6k + 10$

$-\frac{12}{2}k + \frac{20}{2} = -6k + 10$

$-\cancel{6k} + 10 = -\cancel{6k} + 10$

$+6k \qquad +6k$

$10 = 10$

$\boxed{\text{Thus, there are infinitely many solutions to this system}}$

④ $P = 9 - 4Q_D$, $Q_s = 11 - 9P$

Find P and Q_D.

$P = 9 - 4Q_D \rightarrow$ Solve for Q_D

$-9 \quad -9$

$\frac{P-9}{-4} = \frac{4Q_D}{-4}$

$\frac{-P+9}{4} = Q_D$

Set Q_D and Q_s equal to eachother to find equilibrium

$Q_D = 4\left(\frac{-P+9}{4}\right) = (11 - 9P)4 = Q_s$

$-P + 9 = 44 - 36P$

$-9 \qquad -9$

$-P = 35 - 36P$

$-P = 35 - 36P$

$+36P \qquad +36P$

$\frac{35P}{35} = \frac{35}{35}$

$P = 1 \rightarrow \boxed{\text{equilibrium price} = \$1}$

→

Now plug P into Q_s and Q_D to find the quantities

$P = \$1.00$

$Q_s = 11 - 9P \longrightarrow Q_s = 11 - 9(P)$

$= 11 - 9(1)$

$= 11 - 9$

$Q_s = 2$

$P = 9 - 4Q_D$

$\downarrow$

$P = 9 - 4Q_D$

$1 = 9 - 4Q_D$

$-9 \quad -9$

$\frac{-8}{-4} = \frac{-4Q_D}{-4}$

$Q_D = 2$

$\boxed{\text{Therefore, the equilibrium price, P, is } \underline{\$1.00}\text{, The Quantity supplied, } Q_s\text{, is } \underline{2}\text{, and The Quantity demand, } Q_s\text{, is } \underline{2}.}$

⑤ $P = 9 - 4Q_D$ $\quad P = \$3$

$Q_S = 11 - 9P$

$3 = 9 - 4Q_D$
$-9 \quad -9$

$\frac{-6}{-4} = \frac{-4Q_D}{-4}$

$\frac{3}{2} = Q_D$

$Q_S = 11 - 9(3)$
$= 11 - 27$
$Q_S = -16$

$\boxed{Q_D \neq Q_S \rightarrow \text{No, MARKET is not in equilibrium}}$

⑥ x: amount invested at 8%
y: amount invested at 5%] $x + y = 1500$ (Total)

$\underbrace{.08x}_{x} + \underbrace{(1500 - x).05}_{y} = 99$

$.08x + 75 - .05x = 99$

$.03x + 75 = 99$
$-75 \quad -75$

$\frac{.03x}{.03} = \frac{24}{.03}$ $\quad x = 800$

$x + y = 1500$
Plug in $x = 800$, solve for y

$(800) + y = 1500$
$-800 \quad -800$

$y = 700$

$\boxed{\begin{array}{l}\$800 \text{ invested at } 8\% \\ \$700 \text{ invested at } 5\%\end{array}}$

⑦ Let x = Test 1 → 86
$x - 12$: Test 2 → $86 - 12 = 74$

$\frac{x + (x-12)}{2} = 80$

$\frac{x + x - 12}{2} = 80$

$\cancel{2}\left(\frac{2x - 12}{\cancel{2}}\right) = 80(2)$

$2x - 12 = 160$
$+12 \quad +12$

$\frac{2x}{2} = \frac{172}{2}$

$x = 86$

$\boxed{\text{Scores: } 86 \text{ and } 74}$

⑧ Let $x \rightarrow$ correct (3 pts)

$40-x \rightarrow$ wrong (2 points deducted from 3 = 1 pt.)

$$3x-(40-x)=85$$
$$3x-40+x=85$$
$$4x-40=85$$
$$+40 \quad +40$$
$$\frac{4x}{4}=\frac{125}{4}$$
$$x=31.25$$

$\boxed{\begin{array}{l}31.25 \text{ correct}\\ 8.75 \text{ wrong}\end{array}}$

$40-31.25=8.75$

⑨ $(56, 80) \rightarrow (100,100)$

$m=\frac{100-80}{100-56}=\frac{5}{11}$

$y=\frac{5}{11}x+b$

$y=\frac{5}{11}(100)+b \Rightarrow b=\frac{600}{11}$

$y=\frac{5}{11}x+\frac{600}{11}$

If 60 is the lowest passing mark,

$60=\frac{5}{11}x+\frac{600}{11} \Rightarrow x=12$

So, a passing score on the original scale is 12.

⑩ 81 students

for every 8 students, 3 a chaperone

$8\overline{)81}$ = 10.25

Groups of students:

8 8 8 8 8 8 8 8 8 = 72 students

$$\begin{array}{r}81\\-72\\\hline 9\end{array}$$

$\boxed{\text{72 students, 9 chaperones}}$

(11) Calcium : $w = 3y$
Hydrogen : $3x = 2z$
Phosphorus : $x = 2y$
Oxygen : $4x = 8y$

$w = 3y$
$x = 2y$
$z = 3y$

Hydrogen : $3x = 2z$
$3(2y) = 2z$
$6y = 2z$
$3y = z$, so $w = z$

We want $w + x + y + z$ to be the <u>smallest</u> possible.

$w + 2y + y + z$
$\underbrace{3y + 2y} + y + 3y$
$5y + y + 3y$
$6y + 3y$
$= 9y$ ← Must make minimum, so set $y = 1$.

$\therefore$ $w = 3(1) = 3$
$x = 2(1) = 2$
$y = 1$
$z = 3(1) = 3$