

Summary of Curve Sketching

Solutions:

1. $f(x) = -x^3 + 3x - 2$

Domain: $(-\infty, \infty)$

Symmetry: none

x-intercepts: $(-2, 0)$, $(1, 0)$

y-intercept: $(0, -2)$

Asymptotes: none

$$f'(x) = -3x^2 + 3$$

f is increasing on $(-1, 1)$

f is decreasing on $(-\infty, -1) \cup (1, \infty)$

$(-1, -4)$ is a local minimum

$(1, 0)$ is a local maximum

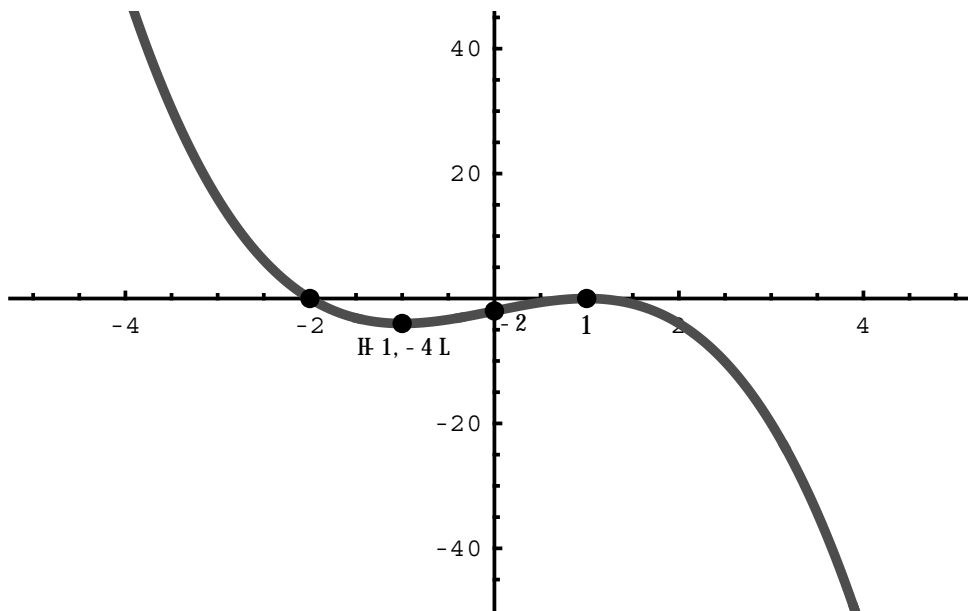
$$f''(x) = -6x$$

f is concave up on $(-\infty, 0)$

f is concave down on $(0, \infty)$

$(0, -2)$ is an inflection point

$$f(x) = -x^3 + 3x - 2$$



2. $g(x) = x^{\frac{5}{3}} + 5x^{\frac{2}{3}}$

Domain: $(-\infty, \infty)$

Symmetry: none

x-intercepts: $(0, 0)$, $(5, 0)$

y-intercept: $(0, 0)$

Asymptotes: none

3. $h(x) = \sin x + \cos x$

Domain: $[-\frac{\pi}{2}, \pi]$

Symmetry: none

x-intercepts: $(-\frac{\pi}{4}, 0), (\frac{3\pi}{4}, 0)$

y-intercept: $(0, 1)$

Asymptotes: none

$h'(x) = \cos x - \sin x$

h is increasing on $[-\frac{\pi}{2}, \frac{\pi}{4}]$

h is decreasing on $(\frac{\pi}{4}, \pi]$

$(\frac{\pi}{4}, \sqrt{2})$ is a local maximum

$(\frac{\pi}{4}, \sqrt{2})$ is the absolute maximum

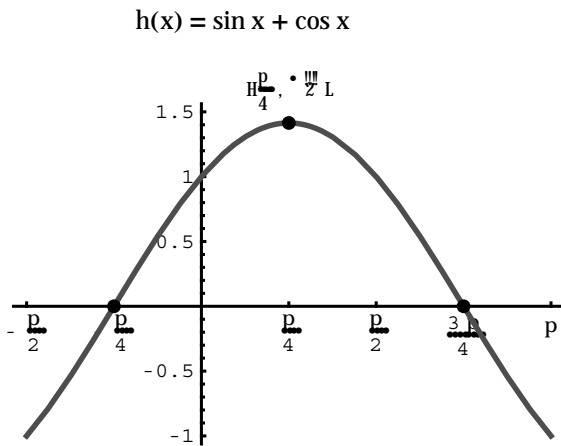
$(-\frac{\pi}{2}, -1)$ and $(\pi, -1)$ are the absolute minima

$h''(x) = -\sin x - \cos x$

h is concave up on $[-\frac{\pi}{2}, -\frac{\pi}{4}] \cup (\frac{3\pi}{4}, \pi]$

h is concave down on $(-\frac{\pi}{4}, \frac{3\pi}{4})$

$(-\frac{\pi}{4}, 0), (\frac{3\pi}{4}, 0)$ are inflection points



4. $f(x) = \frac{-x^2+x+2}{(x-1)^2}$

Domain: $(-\infty, 1) \cup (1, \infty)$

Symmetry: none

x-intercepts: $(-1, 0), (2, 0)$

y-intercept: $(0, 2)$

vertical asymptote: $x = 1$

horizontal asymptote: $y = -1$

$f'(x) = \frac{x-5}{(x-1)^3}$

f is increasing on $(-\infty, 1) \cup (5, \infty)$

f is decreasing on $(1, 5)$

$(5, -\frac{9}{8})$ is a local minimum

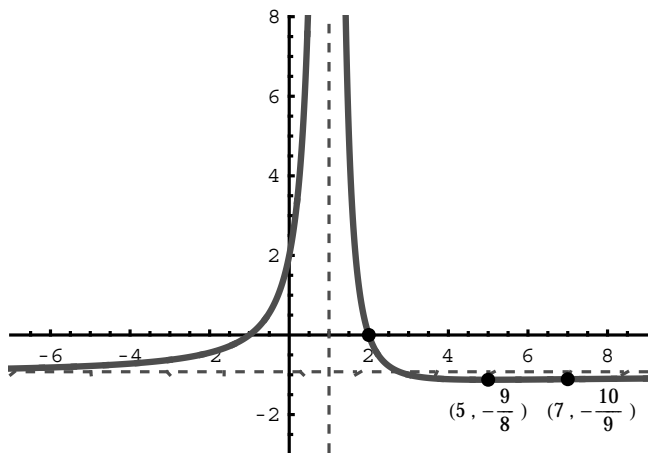
$f''(x) = \frac{-2x+14}{(x-1)^4}$

f is concave up on $(-\infty, 1) \cup (1, 7)$

f is concave down on $(7, \infty)$

$(7, -\frac{10}{9})$ is an inflection point

$f(x) = \frac{-x^2+x+2}{(x-1)^2}$



5. Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

Symmetry: origin

x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

vertical asymptotes: $x = -2, x = 2$

slant asymptote: $y = \frac{x}{2}$

$$y' = \frac{2x^4 - 24x^2}{(2x^2 - 8)^2}$$

y is increasing on $(-\infty, -2\sqrt{3}) \cup (2\sqrt{3}, \infty)$

y is decreasing on $(-2\sqrt{3}, -2) \cup (-2, 2) \cup (2, 2\sqrt{3})$

$(-2\sqrt{3}, -2.6)$ is a local maximum

$(2\sqrt{3}, 2.6)$ is a local minimum

$$y'' = \frac{8x(4x^2 + 48)}{(2x^2 - 8)^3}$$

y is concave up on $(-2, 0) \cup (2, \infty)$

y is concave down on $(-\infty, -2) \cup (0, 2)$

$(0, 0)$ is an inflection point

