Solutions:

1. Verify that the function satisfies the three hypotheses of the Mean Value Theorem (MVT) on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

   a) $c = 1$

   b) $c = \left(\frac{1}{3}\right)^{\frac{3}{2}}$

   c) $c = \sqrt{3}$

2. Show that the equation $x^3 - 15x + c = 0$ has at most one root in the interval $[-2, 2]$.

3. If $f(1) = 10$ and $f'(x) \geq 2$, for $1 \leq x \leq 4$, how small can $f(4)$ possibly be?

   $\because f'(x) \geq 2$, for $1 \leq x \leq 4$
   $\rightarrow f$ is differetitiable on $[1, 4]$
   $\rightarrow f$ is continuous on $[1, 4]$
   $\rightarrow f'(x) = \frac{f(4)-f(1)}{4-2} \wedge f'(x) \geq 2$ [By MVT and Assumption]
   $\rightarrow \frac{f(4)-f(1)}{4-2} \geq 2$
   $\rightarrow f(4) \geq 2(4-2) + f(1) = 4 + 10 = 14$
   $\therefore f(4) \geq 14$

4. Does there exist a function f such that $f(0) = -1$, $f(2) = 4$ and $f'(x) \leq 2$ for all x?