Workshop Exercises: Techniques of Integration I (Solutions)

1. Calculate the anti-derivatives.

   a. $x^5 + c$

   b. $\frac{3x^4}{4} + x^3 - 7x^2 - 9x + c$

   c. $-\cos(x) + c$

   d. $\sin(x) + c$

   e. $\frac{2x^{\frac{3}{2}}}{3} + c$

   f. $\frac{2x^{\frac{3}{2}}(3x-15)}{15} + c$

2. Consider the function $f(x) = x^2$ defined on $[1, 3]$.

   a. Find the Riemann sum of f for the partition $\{1, 1.2, 1.9, 2.4, 3\}$ choosing right-hand endpoints as sample points.

   $$11.095$$

   b. Write the expression for the Riemann sum of f on $[1, 3]$ with n equal subintervals using right-hand endpoints.

   $$\frac{2}{n}\sum_{I=1}^{n}\left(1 + \frac{4i}{n} + \frac{4i^2}{n^2}\right)$$

   c. Find $\int_1^3 x^2\, dx$ using the above formulas.

   $$\frac{26}{3}$$

3. Compute the left- and right-hand approximations to $\int_1^6 2x^3\, dx$ with $n = 4$ subdivisions.

   Left-hand approximations: 406.0938

   Right-hand approximations: 943.5938