Workshop Exercises: Differentiation II

1. Differentiate the following functions.
   a) $f(x) = e^{2x}$.
   b) $h(x) = e^{-3x\csc(x)}$.
   c) $h(x) = 3^{1/x}$.
   d) $f(x) = (\cos(x)) \cdot 2^{-4x}$.
   e) $f(x) = \sin^{-1}(x^2)$.
   f) $g(x) = (\tan^{-1}(2x))^3$.
   g) $h(x) = \cos^{-1}(e^{3x})$.
   h) $f(x) = \ln(2x)$.
   i) $h(x) = (\ln(x))^2$.
   j) $f(x) = \log_4 \sqrt{x^2 + 1}$.
   k) $g(x) = \tan^2(\ln(x))$.

2. Use implicit differentiation to find y' (or $\frac{dy}{dx}$) in terms of x and y, where $x^2 - 3x^2y^2 + y^4 = -5$.

3. Find the equation of the line tangent to the curve $y + \cos(xy^2) + 3x^2 = 4$ at the point (1, 0).

4. Differentiate using logarithmic differentiation.
   a) $y = (3x + 2)^3 (4x - 5)^5$.
   b) $y = x^{\cos(x)}$.
   c) $y = x^{\ln(x)}$.
   d) $y = (\ln(x))^x$.

5. An object is moving along a coordinate line according to the formula
   $s(t) = t^3 - 3t^2 - 24t$, where $s(t)$ is the position function (in feet) and t is in second ($t \geq 0$).
   a) Find $v(t)$; find when $v(t) = 0$, and the position(s) of the object when $v(t) = 0$.
   b) Find the interval(s) when the object is moving to the right,
   and the interval(s) when the object is moving to the left.
   c) Find the displacement of the object in the first 5 seconds.
   d) Find the total distance traveled during the first 5 seconds.
   e) Find $a(3)$, the acceleration of the object at $t = 3$.

6. A bacterial population starts with 10,000 bacteria and grows at a rate proportional to its size.
   After 2 hours there are 40,000 bacteria.
   a) Find the number of bacteria after 5 hours.
   b) When will the population reach 1 million?

7. (Carbon Dating) All living things contain carbon-12, which is stable, and carbon-14, which is radioactive. While a plant or animal is alive, the ratio of these two isotopes of carbon remains unchanged since the carbon-14 is constantly renewed; after death, no more carbon-14 is absorbed. The half-life of carbon-14 is 5,730 years. If changed logs of an old ford showed only 70% of the carbon-14 expected in living matter, when did the fort burn down? assume the fort burned soon after it was build of freshly sawn logs.

8. Estimate $(32.3)^{2/5}$ using differentials.

9. Estimate $\tan(44.8\,°)$ using differentials.

10. The side of a cube is measured as 2.4 meters with a possible error of ± 0.0005 meters.
    Find the volume of the cube and an estimate for the possible error for this value.

11. The sides of a cylindrical container of radius 6 feet and height 10 feet are to be coated with waterproofing paint so that the thickness of the paint is 0.02 inches. Use differentials to estimate the number of gallons that are required, where 1 gal ≈ 231 in^3. (Note: the volume of a right circular cylinder is given by $V = \pi r^2 h$. Here, h remains constant since only the sides are painted, so the equation is differentiated with respect to r.