Workshop Exercises: Limits and Continuity

1. Determine whether the limit exists at the indicated point. Justify.

a) $f(x) = \begin{cases} 2x, & x < 1 \\ x^2 + 2, & x \geqslant 1 \end{cases} \quad a = 1.$ b) $g(x) = \begin{cases} 2x - 1, & x < -1 \\ 4, & x = -1 \\ x^3 - 2, & x > -1 \end{cases} \quad a = -1.$

2. Evaluate the limit, if it exists. (Limits can evaluate to ∞ or $-\infty$).

a) $\lim_{x\to 2} \frac{x^2-4x+2}{x+2}$. e) $\lim_{x\to\frac{1}{2}} \frac{2x^2+x-1}{2x^2-5x+2}$. i) $\lim_{x\to 5} \frac{x-5}{2-\sqrt{x-1}}$.

b) $\lim_{x\to 3} \frac{x-3}{x^2-9}$. f) $\lim_{x\to -3} \frac{2x^2+5x-3}{3x^2-4}$. j) $\lim_{x\to 3} \left(\frac{1}{x^2-5x+6} - \frac{1}{x-3}\right)$.

c) $\lim_{x\to 4} \frac{x^2-2x-8}{x^2+2x-24}$. g) $\lim_{x\to 2^+} \frac{3x+2}{x^2-4}$. k) $\lim_{x\to -2} \frac{|x+2|}{x+2}$.

d) $\lim_{x\to 1} \frac{x^2+4x-5}{x^3-1}$. h) $\lim_{x\to 2} \frac{3x+2}{x^2-4}$. l) $\lim_{x\to 0^-} \frac{x}{x-|x|}$.

3. Evaluate the limit, if it exists. (Limits can evaluate to ∞ or $-\infty$).

a) $\lim_{x\to -3^+} \frac{x+2}{x+3}$. b) $\lim_{t\to\infty} \frac{t-t\sqrt{t}}{2t^{3/2}+3t-5}$. c) $\lim_{x\to\infty} \sqrt{9x^2+x} - 3x$.

4. (Optional) Give an $\epsilon - \delta$ proof for the following limits:

a) $\lim_{x\to 1} (2x+1) = 3$. b) $\lim_{x\to 4} (3x-7) = 5$. c) $\lim_{x\to 3} (x^2+x-5) = 7$.

5. Determine if the following functions are continuous at the indicated point. Justify.

a) $f(x) = \begin{cases} 2x - 1, & x < -1 \\ 4, & x = -1 \\ x^3 - 2, & x > -1 \end{cases} \quad a = -1.$ c) $f(x) = \begin{cases} 2x - 1, & x < 1 \\ \cos 2\pi x, & x \geqslant 1 \end{cases} \quad a = 1.$

b) $g(x) = \begin{cases} x^2 + 2, & x < 1 \\ 0, & x = 1 \\ 5x - 2, & x > 1 \end{cases} \quad a = 1.$

6. Use the Squeeze Theorem to show that $\lim_{x\to 0} \sqrt[3]{x} \sin \frac{1}{x^2} = 0$.

7. Use the Intermediate Value Theorem to show that the function
$f(x) = x^3 - 2x^2 + x - 1$ has a root in the interval [1, 2].