Workshop Exercises: The Mean Value Theorem

1. Verify that the function satisfies the three hypotheses of the Mean Value Theorem (MVT) on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

   a) $f(x) = 2x^2 - 3x + 1$, $[0, 2]$.

   b) $f(x) = \sqrt[3]{x-1}$, $[0, 1]$.

   c) $f(x) = \frac{1}{x}$, $[1, 3]$.

2. Show that the equation $x^3 - 15x + c = 0$ has at most one root in the interval $[-2, 2]$.

3. If $f(1) = 10$ and $f'(x) \geq 2$, for $1 \leq x \leq 4$, how small can $f(4)$ possibly be?

4. Does there exist a function f such that $f(0) = -1$, $f(2) = 4$ and $f'(x) \leq 2$ for all x?