Workshop Exercises: Optimization

1. Find the dimensions of the rectangle of maximum area that has vertices on the curve $y = 4 - x^2$ and the x-axis.

2. A poster is to contain 50 square inches of printed matter with margins of 4 inches at the top and bottom of the page and 2 inches at each side. Find the overall dimensions if the total area is to be a minimum.

3. Find the dimensions of the cylinder of maximum volume that can be inscribed in a cone that has height 8 inches and radius 6 inches.

4. A motorist is in a desert in a jeep. Directly east and $4\sqrt{2}$ miles from the motorist is Town A which is connected by a straight road to Town B, which is 10 miles south from Town A. If the motorist can drive 15 miles per hour in the desert and 45 miles per hour on the road, where should he intersect the road if he wishes to reach Town B in the shortest time?

5. A wire of length 3 feet is cut into 2 pieces, one being bent to form a square and the other to form an equilateral triangle. How should the wire be cut if the sum of the two areas is to be a) a minimum or b) a maximum? (You can also use the wire entirely for the square or for the triangle).

6. A car rental agency has 24 identical cars. The owner of the agency finds that at a price of \$10 per day, all the cars can be rented. However, for each \$1 increase in rental price, one of the cars is not rented. How much should he charge to maximize the income of the agency?

7. Show that the rectangle with maximum area that can be inscribed in an ellipse centered at the origin is $2\,a\,b$, where the equation of the ellipse is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.