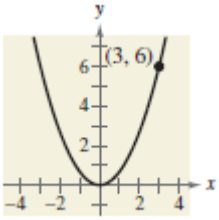
PRECALCULUS PROBLEM SESSION #14- PRACTICE PROBLEMS

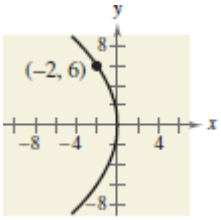
Parabolas

A) Find the vertex, focus, and directrix of the parabolas and sketch its graph.

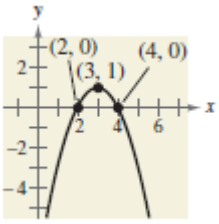
1. $y = \frac{1}{2}x^2$
2. $y^2 = -6x$
3. $x^2 + 6y = 0$
4. $(x-1)^2 + 8(y+2)$
5. $(x+5) + (y-1)^2 = 0$
6. $\left(x + \frac{3}{2}\right)^2 = 4(y-2)$
7. $\left(x + \frac{1}{2}\right)^2 = 4(y-1)$
8. $y = \frac{1}{4}(x^2 - 2x + 5)$
9. $x = \frac{1}{4}(y^2 + 2y + 33)$
10. $y^2 + 6y + 8x + 25 = 0$
11. $y^2 - 4y - 4x = 0$

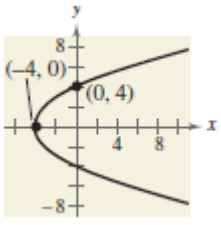
B) Find the standard form of the equation of the parabola with its vertex at the origin.

1. Focus: $(0, -\frac{3}{2})$
2. Focus: $(-2,0)$
3. Directrix: $y = -1$
4. Directrix: $x = 2$
5. Horizontal axis and passes through the point $(4,6)$
6. 

7. 

C) Find the standard form of the equation of the parabola.

1. Vertex: $(5, 2)$; Focus: $(3,2)$
2. Vertex: $(0,4)$; Directrix: $y = 2$
3. Focus: $(2,2)$; Directrix: $x = -2$
4. 

5. 

D) Word problem

1. Each cable of a suspension bridge is suspended (in the shape of a parabola) between two towers that are 120 meters apart and whose tops are 20 meters about the roadway. The cables touch the roadway midway between the towers.
    a. Create a sketch of the bridge. Draw a rectangular coordinate system on the bridge with the center of the bridge at the origin. Identify the coordinates of the known points.
    b. Find an equation for the parabolic shape of each cable.

c. Complete the table by finding the heights y of the suspension cables over the roadway at distances of x meters from the center of the bridge

x	0	20	40	60
y				

Ellipses

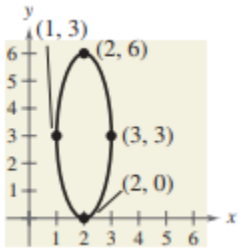
A) Find the center, vertices, foci, and eccentricity of the ellipse, and then sketch the graph.

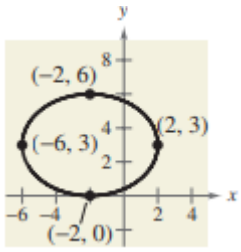
1. $\frac{x^2}{25}+\frac{y^2}{16}=1$
2. $\frac{x^2}{5}+\frac{y^2}{9}=1$
3. $\frac{(x+3)^2}{16}+\frac{(y-5)^2}{25}=1$
4. $\frac{(x+5)^2}{\frac{9}{4}}+(y-1)^2=1$
5. $6x^2+2y^2+18x-10y+2=0$
6. $16x^2+25y^2-32x+50y+16=0$

B) Find the standard form of the equation of the ellipse with center at the origin.

1. Vertices: $(\pm 6,0)$; Foci: $(\pm 2,0)$
2. Foci: $(\pm 5,0)$; Major axis of length 12
3. Vertices: $(0,\pm 5)$; Passes through the point $(4,2)$

C) Find the standard form of the equation of the specified ellipse.

1. 

2. 

3. Vertices: $(0,4)$, $(4,4)$; Minor axis of length 2
4. Center: $(0,4)$; $a=2c$; Vertices: $(-4,4)$, $(4,4)$

Hyperbolas

A) Find the center, vertices, foci, and the equations of the asymptotes of the hyperbola, and sketch its graph.

1. $x^2-y^2=1$
2. $\frac{y^2}{25}-\frac{x^2}{81}=1$
3. $\frac{(y+6)^2}{\frac{1}{9}}-\frac{(x-2)^2}{\frac{1}{4}}=1$
4. $9x^2-y^2-36x-6y+18=0$
5. $x^2-9y^2+2x-54y-80=0$
6. $\frac{(x-1)^2}{4}-\frac{(y+2)^2}{1}=1$

B) Find the standard form of the equation of the specified hyperbola with the center at the origin.

1. Vertices: $(0, \pm 2)$; Foci: $(0, \pm 4)$
2. Vertices: $(\pm 1, 0)$; Asymptotes: $y = \pm 5x$
3. Foci: $(0, \pm 8)$; Asymptotes: $y = \pm 4x$

C) Find the standard form of the equation of the specified hyperbola.

1. Vertices: $(4,1), (4,9)$; Foci: $(4,0), (4,10)$
2. Vertices: $(-2,1), (2, 1)$; Passes through the point $(5,4)$
3. Vertices: $(0, 4), (0, 0)$; Passes through the point $\left(\sqrt{5}, -1\right)$
4. Vertices: $(1, 2), (3,2)$; Asymptotes: $y = x,\ y = 4 - x$
5. Vertices: $(0, 2), (6, 2)$; Asymptotes: $y = \frac{2}{3}x,\ y = 4 - \frac{2}{3}x$

D) Word Problem

1. A hyperbolic mirror (used in some telescopes) has the property that a light ray directed at a focus will be reflected to the other focus (see figure). The focus of a hyperbolic mirror has coordinates $(24, 0)$. Find the vertex of the mirror if its mount has coordinates $(24, 24)$.