PRECALCULUS PROBLEM SESSION #11

Inverse Trigonometric Functions

1. Find the exact value of each expression (1)

   a. $\sin^{-1}\frac{\sqrt{3}}{2}$ b. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ c. $\cos^{-1} 1$ d. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

2. Find the exact value of each expression, if possible. Do not use a calculator. (2)

   a. $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)$ b. $\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$ c. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

3. Why is it that $\sin\frac{5\pi}{6} = \frac{1}{2}$, but $\sin^{-1}\left(\frac{1}{2}\right) \neq \frac{5\pi}{6}$? (3)

4. Use a sketch to find the exact value of each expression:

   a. $\cot\left(\sin^{-1}\frac{5}{13}\right)$ b. $\sec\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$

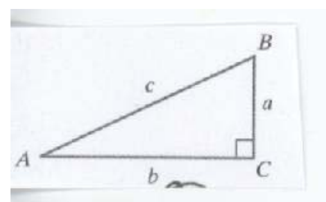
5. Use a right triangle to write each expression as an algebraic expression. Assume that *x* is positive and that the given inverse trigonometric function is defined for the expression in *x*.

   a. $\sin(\cos^{-1} 2x)$ b. $\sec\left(\cos^{-1}\frac{1}{x}\right)$

6. Explain in your own words why the ranges of the inverse trigonometric functions are restricted.

Applications of Trigonometric Functions

1. Solve the right triangle shown. Round to two decimal places and express angles to the nearest tenth of a degree.
   A = 41.5°, b = 20

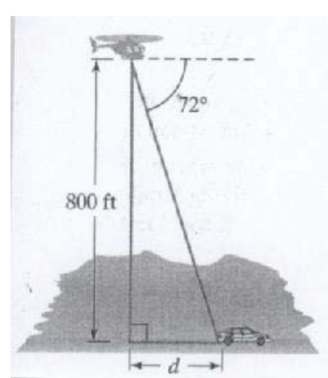


2. An object is attached to a coiled spring. The object is pulled down (negative direction from the rest position) and then released. Write an equation for the distance of the object from its rest position after t seconds when the distance from rest position at t = 0 is 8 inches, the amplitude is 8 inches, and the period is 2 seconds.
3. An object moves in simple harmonic motion described by the given equation, where t is measured in seconds and d in inches. Find the following: a) the maximum displacement, b)the frequency, and c) the time required for one cycle.

   a. $d = -8\cos\frac{\pi}{2}t$

4. From a point on level ground 30 yards from the base of a building, the angle of elevation is 38.7°. Approximate the height of the building to the nearest foot.

5. A police helicopter is flying at 800 feet. A stolen car is sighted at an angle of depression of 72°. Find the distance of the stolen car, to the nearest foot, from a point directly below the helicopter.



Selected problems were taken from Blitzer's PreCalculus

6. A building that is 250 feet high casts a shadow 40 feet long. Find the angle of elevation, to the nearest tenth of a degree, of the sun at this time.

7. A flagpole is situated on top of a building. The angle of elevation from a point on level ground 330 feet from the building to the top of the flagpole is 63°. The angle of elevation from the same point to the bottom of the flagpole is 53°. Find the height of the flagpole to the nearest tenth of a foot.

Verifying Trigonometric Identities

1. Derive the second and third Pythagorean identities from the first (*Hint: divide through by the square of the appropriate trigonometric function)

2. What is the difference between a trigonometric equation that is an identity and a trigonometric equation that is not an identity? Give an example of each.

3. Verify each identity:

a) $\cos x \; \csc x = \cot x$ b) $\csc x - \csc x \cos^2 x = \sin x$ c) $\frac{\cos\theta \sec\theta}{\cot\theta} = \tan\theta$

d) $\cos t \; \cot t = \frac{1-\sin^2 t}{\sin t}$ e) $\frac{\cot^2 t}{\csc t} = \csc t - \sin t$ f) $\frac{1-\sin\theta}{\cos\theta} = \sec\theta - \tan\theta$

g) $1 - \frac{\cos^2 x}{1+\sin x} = \sin x$ h) $\frac{\sin x}{\cos x+1} + \frac{\cos x-1}{\sin x} = 0$ i) $\frac{\csc x-\sec x}{\csc x+\sec x} = \frac{\cot x-1}{\cot x+1}$

j) $\frac{\csc t-1}{\cot t} = \frac{\cot t}{\csc t+1}$ k) $\frac{\cos^2 t+4\cos t+4}{\cos t+2} = \frac{2\sec t+1}{\sec t}$

Sum and Distance Formulas

1. Use one or more of the six sum and difference identities to solve. Find the exact value of each expression:

a. $\sin 75°$ b. $\cos 105°$ c. $\tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

2. Write each expression as the sine, cosine, or tangent of an angle. Then find the exact value of the expression.

a. $\frac{\tan 50° - \tan 20°}{1+\tan 50° \tan 20°}$ b. $\sin\frac{7\pi}{12}\cos\frac{\pi}{12} - \cos\frac{7\pi}{12}\sin\frac{\pi}{12}$

3. Show that $\cos(A-B) + \cos(A+B) = 2\cos A \cos B$

4. Find the exact value under the given conditions: a) $\cos(\alpha+\beta)$ b) $\sin(\alpha+\beta)$ c) $\tan(\alpha+\beta)$

a) $\sin\alpha = \frac{4}{5}$, α lies in quadrant I, and $\sin\beta = \frac{7}{25}$, β lies in quadrant II

b) $\tan\alpha = -\frac{4}{3}$, α lies in quadrant II, and $\cos\beta = \frac{2}{3}$, β lies in quadrant I

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