PRECALCULUS PROBLEM SESSION #12

Double-Angle, Power-Reducing, and Half-Angle Formulas

1. Use the information to find the exact value of each of the following: a) $\sin 2\theta$ b) $\cos 2\theta$ c) $\tan 2\theta$
   i) $\sin\theta = \frac{12}{13}$, θ lies in quadrant II ii) $\cot\theta = 3$, θ lies in quadrant III

2. Use a half-angle formula to find the exact value of each expression:
   a. $\cos 22.5°$ b. $\sin 105°$ c. $\tan\frac{3\pi}{8}$

3. Use the given information to find the exact value of each of the following: a) $\sin\frac{\propto}{2}$ b) $\cos\frac{\propto}{2}$ c) $\tan\frac{\propto}{2}$
   a. $\tan\alpha = \frac{8}{15}, 180° < \alpha < 270°$ **b.** $\mathbf{sec}\,\boldsymbol{\alpha} = \mathbf{-3}, \frac{\boldsymbol{\pi}}{\mathbf{2}} < \alpha < \pi$

4. Use the power-reducing formula to rewrite the expression as an equivalent expression that does not contain powers of trigonometric functions greater than 1. $10\cos^4 x$

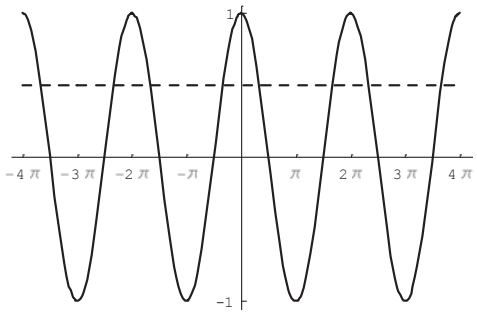
5. Find all the errors in the following and then verify that this solution is NOT an identity by plugging in $x = \pi$

$$2\sin^2 2x + \cos 4x$$
$$= 2(2\sin x\cos x)^2 + 2\cos 2x$$
$$= 8\sin^2 x\cos^2 x + 2(\cos^2 x + \sin^2 x)$$
$$= 8\sin^2 x\cos^2 x + 2$$

6. Explain how $\cos 2A = 2\cos^2 A - 1$ and $\cos 2A = 1 - 2\sin^2 A$ can both be derived from $\cos 2A = \cos^2 A - \sin^2 A$.

Trigonometric Equations

1. We are interested in solutions to $\cos x = 0.5$. The figure below shows a partial graph of both sides of the equation:
   (a) How many solutions does the equation have on the interval $[0, 2\pi)$? What are they?
   (b) How many solutions does the equation have on the interval $(-\infty, \infty)$? Discuss a method of writing all solutions to the equation.



2. Find all solutions of each equation:
   a. $\cos x = \frac{\sqrt{3}}{2}$ b. $\tan x = \sqrt{3}$ c. $2\sin x + \sqrt{3} = 0$ d. $\cos 2x = \frac{\sqrt{2}}{2}$

3. Solve each equation on the interval $[0,2\pi)$:
   a. $\cos 2x = \frac{\sqrt{2}}{2}$ b. $\tan\frac{x}{2} = \frac{\sqrt{3}}{3}$ c. $2\sin^2 x + \sin x - 1 = 0$
   d. $\cos^2 x + 2\cos x - 3 = 0$ e. $3\tan^2 x - 9 = 0$

4. Does $5\sin x = 7$ have a solution for x? Why or why not?

5. Jan lists her answer to a problem as $\frac{\pi}{6} + k\pi$ for any integer k, while Jacob lists his answer as $\frac{\pi}{6} + (2\pi)k$ and $\frac{7\pi}{6} + (2\pi)k$, for any integer k. Are their answers equivalent? Why or why not?

6. Use an identity to solve the equation $2\cos^2 x - \sin x - 1 = 0$ on the interval $[0,2\pi)$.

Selected problems were taken from Blitzer's PreCalculus