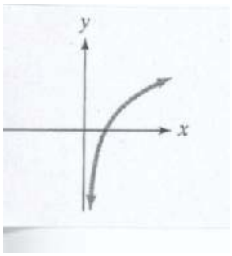
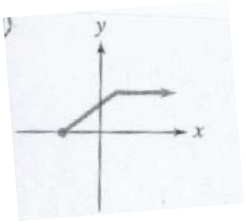
PRECALCULUS PROBLEM SESSION #3

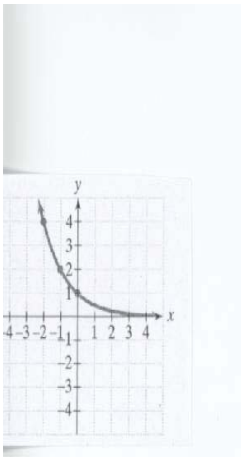
Inverse Functions

1. Find $f(g(x))$ and $g(f(x))$ and determine if each pair of functions f and g are inverses of each other.
   $f(x) = 4x + 9$ and $g(x) = \frac{x-9}{4}$
2. Find an equation for $f^{-1}(x)$, the inverse function, and verify that your equation is correct by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
   a) $f(x) = x^3 - 1$ b) $f(x) = \frac{4}{x} + 9$ c) $f(x) = \frac{2x-3}{x+1}$ d) $f(x) = 3x - 1$
3. Which graphs represent functions that have inverse functions?

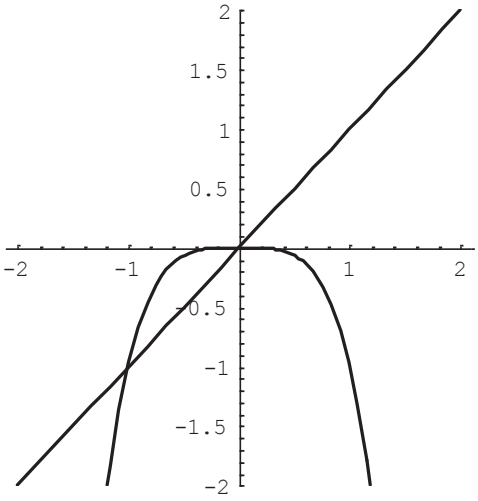


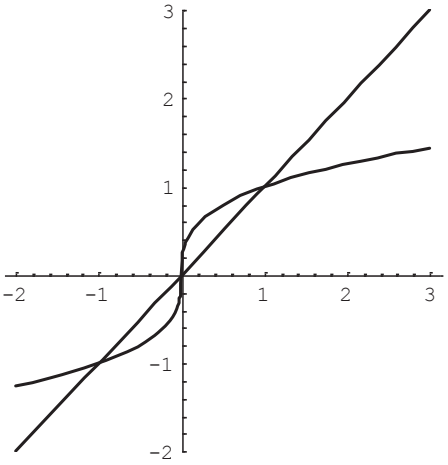


4. Use the graph shown to draw a graph of the the inverse function



5. Sketch the inverse of each of the functions graphed below (for convenience, the line *y* = *x* is included):





6. Find an equation for $f^{-1}(x)$, graph f(x) and $f^{-1}(x)$ on the same graph, and use interval notation to give the domain and range of f and f^{-1}. $f(x) = x^2 - 1, x \leq 0$.

7. The function $f(x) = x^2 - 3$ is not one-to-one. Restrict the domain of *f* so that its inverse is a function. Find the inverse and state the restriction on the domain of its inverse.

8. To find the inverse of *y* = *f*(*x*), the variables *x* and *y* are interchanged. Explain the effect of interchanging *x* and *y* on the graph of *f*(*x*).

Selected problems were taken from Blitzer's PreCalculus

9. Show that $f\left[f^{-1}(a)\right]=a$ given that $f(x) = mx + b$.

10. Given $f(x) = mx + b$, find the slope and y-intercept of the inverse function.

11. If a relation is not a function, is it possible for its inverse to be a function? If so, give an example.

12. Describe the difference between $f^{-1}(x)$ and $[f(x)]^{-1}$

Distance and Midpoint Formulas and Circles

1. Find the midpoint of the line segment with endpoints $(-\frac{2}{5},\frac{7}{15})$ and $\left(-\frac{2}{5},-\frac{4}{15}\right)$

2. Describe geometrically the set of all points (x, y) that are equidistant from the points (1,1) and (3, 0), and then use the distance formula to verify your result algebraically.

3. Find the distance between $(0,2)$ and $(4,3)$.
4. Give the center and radius of the circle described by the equation $(x+1)^2+(y-4)^2=25$. Graph and tell the domain and range.

5. Complete the square and write the equation in standard form. Then give the center and radius of each circle and graph the equation.
   a) $x^2+y^2+8x+4y+16=0$ b. $x^2+y^2-4x-12y-9=0$

Real World Problems

1. A car rental agency charges \$180 per week plus \$0.25 per mile to rent a car. Express the weekly cost to rent the car, $\boldsymbol{f}$, as a function of the number of miles driven during the week, $\boldsymbol{x}$. How many miles did you drive during the week if the weekly cost to rent the car was \$395?

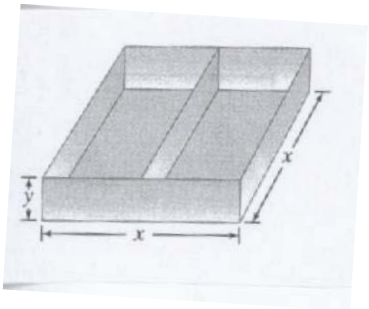
2. An open box is made from a square piece of cardboard 30 inches on a side by cutting identical squares from the corners and turning up the sides. (a) Express the volume of the box, $\boldsymbol{V}$, as a function of the length of the side of the square cut from each corner, $\boldsymbol{x}$. (b) Find and interpret $\boldsymbol{V(3)}$, $\boldsymbol{V(4)}$, $\boldsymbol{V(5)}$, $\boldsymbol{V(6)}$, and $\boldsymbol{V(7)}$. (c) What is happening to the volume of the box as the length of the side of the square cut from each corner increases? (d) Find the domain of $\boldsymbol{V}$.

3. You have 600 feet of fencing to enclose a rectangular field. However, one side of the field lies along a canal and requires no fencing. Express the area of the field, $\boldsymbol{A}$, as a function of one its dimensions, $\boldsymbol{x}$.

4. The figure shows an open box with a square base and a partition down the middle. The box is to have a volume of 400 cubic inches. Express the amount of material needed to construct the box, $\boldsymbol{A}$, as a function of the length of a side of its square base, $\boldsymbol{x}$.

5. Let $P(x,y)$ be a point on the graph of $y=\sqrt{x}$. Express the distance, d, from P to $(2,0)$ as a function of the point's x-coordinate.

6. Let $P(x,y)$ be a point on the graph of $y=x^2-8$ Express the distance, d, from P to the origin as a function of the point's x-coordinate.



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