PRECALCULUS PROBLEM SESSION #4

Complex Numbers

1. Simplify and write in standard form:

   a) $5i^5 - 3i^2 + i$ b) $2i^{16} + i^{22}$ c) $(-7 + 5i) - (-9 - 11i)$ d) $(2 + 7i)(2 - 7i)$

   e) $\frac{-12+\sqrt{-28}}{32}$ f) $(-4 - 8i)(3 + i)$ g) $\frac{-6i}{3+2i}$

2. Write each of the following in the standard form: a) $\frac{1+4i}{3-2i}$ b) $\frac{2-3i}{4-3i}$

3. Solve using the quadratic formula
   a) $2x^2 + 2x + 3 = 0$ b) $3x^2 = 4x - 6$

4. When will the equation $ax^2 + bx + c = 0$ have complex solutions with non-zero imaginary parts?

Quadratic Functions

1. Why is the vertex of a parabola called a maximum or a minimum?

2. Given $f(x) = x^2$, which of the following represent graphs that are reflections about the x-axis for $f(x)$?

   a) $f(x) = -x^2$ b) $f(x) = (-x)^2$ c) $f(x) = -(x - 2)^2$

3. Sketch the graph using the vertex and intercepts. Give the equation of the parabola's axis of symmetry and determine the domain and range.
   a) $f(x) = 6 - 4x + x^2$ b) $f(x) = (x - 1)^2 - 2$ c) $f(x) = x^2 - 2x - 15$

4. Without graphing, determine the function's domain and range, and find the function's minimum or maximum value, if applicable: $f(x) = -2x^2 - 12x + 3$.

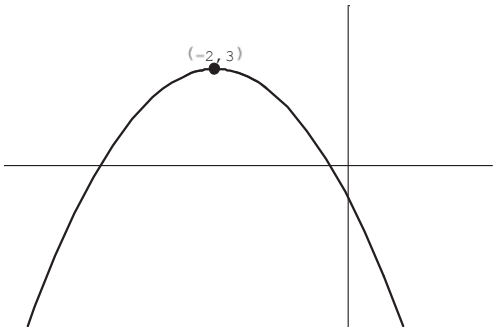
5. Why does the general form of a quadratic function $y = ax^2 + bx + c$ contains the restrictions $a \neq 0$?

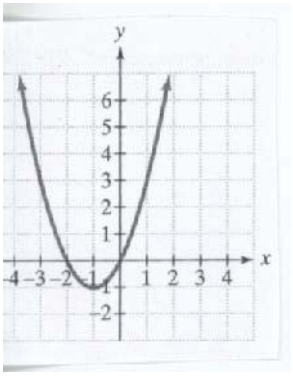
6. What attribute(s) do the equations $y = ax^2 + c$ and $y = ax^2$ have in common?

7. a) Among all pairs of numbers whose difference is 24, find a pair whose product is as small as possible. What is the minimum product?
   b) Among all pairs of numbers whose sum is 20, find a pair whose product is as large as possible. What is the maximum product?

Selected problems were taken from Blitzer's PreCalculus

8. Write the equations represented by the graphs below.





9. For t in seconds, the height of a baseball in feet is given by the formula

$$y = f(t) = -16t^2 + 64t + 3.$$

Using algebra, find the maximum height reached by the baseball and the time at which the ball reaches the ground.

10. You have 80 yards of fencing to enclose a rectangular region. Find the dimensions of the rectangle that maximize the enclosed area. What is the maximum area?

11. A rectangular dog pen is to be made with 100 feet of fence wire.

    (a) If x represents the width of the pen, express its area A(x) in terms of x.

    (b) Considering the physical limitations, what is the domain of the function A?

    (c) Graph the function for this domain.

    (d) Determine the dimensions of the rectangle that will make the area maximum.

    (e) Rework the problem with the added assumption that an existing property fence will be used for one side of the pen.

12. A rectangular playground is to be fenced off and divided in two by another fence parallel to one side of the playground. Four hundred feet of fencing is used. Find the dimensions of the playground that maximize the total enclosed area. What is the maximum area?

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