PRECALCULUS PROBLEM SESSION #5

Polynomial Functions and Their Graphs

1. Are the functions in (a) through (f) polynomials? If so, of what degree?

   a) $y = 4x^2 + 2$ b) $y = 5^x - 2$ c) $y = 5 + x$

   d) $y = 4x^4 - 3x^3 + 2e^x$ e) $f(x) = 7x^2 + 9x^4$ f) $h(x) = 8x^3 - x^2 + \frac{2}{x}$

2. Use the Leading Coefficient Test to find out the graph's end behavior. Find the x-intercepts and y-intercepts. Determine whether the graph as y-axis symmetry, origin symmetry, or neither.
   a) $f(x) = x^4 - x^2$ b) $f(x) = -2(x-4)^2(x^2-25)$

3. Describe in words the long-run behavior as $x \to \infty$ of the following functions in questions a-d. What power function does each resemble?

   (a) $y = 16x^3 - 430x^2 - 2$ (b) $y = 4x^4 - 2x^2 + 3$

   (c) $y = 3x^3 + \dfrac{2x^2}{x^{-7}} - 7x^5 + 2$ (d) $y = \dfrac{5x^2}{x^{\frac{3}{2}}} + 2$

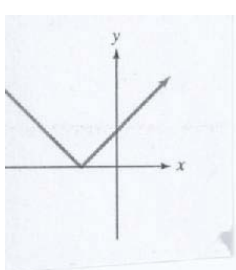
4. Use the Intermediate Value Theorem to show that the polynomial has a real zero between the given integers.
   (a) $f(x) = x^4 + 6x^3 - 18x^2$ between 2 and 3 (b) $f(x) = x^3 - 4x^2 + 2$ between 0 and 1

5. Find the zeros for the function $f(x) = 2(x+5)(x+2)^2$. What is the multiplicity for each zero? Does the graph cross the x-axis, or touch the x-axis and turns around, at each zero? Then graph the polynomial. Use the Leading Coefficient Test to determine the graph's end behavior. Find the y-intercept. Determine whether the graph has y-axis symmetry, origin symmetry, or neither. And, if necessary, find a few additional points and graph the function. Use the maximum number of turning points to check whether it is drawn correctly.

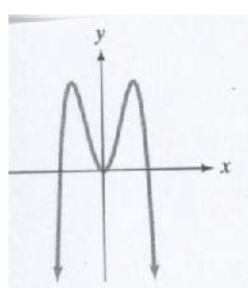
6. If *r* is a real zero of a polynomial *P*(*x*) with real coefficients, then *r* is also an *x* intercept for the graph of *P*(*x*). Discuss the difference between the graph of *P*(*x*) at a real zero of odd multiplicity and at a real zero of even multiplicity.

7. Which graphs are not polynomial functions?

   a.



   b.



8. Graph: $f(x) = -x^2(x+2)(x-2)$

Selected problems were taken from Blitzer's <u>PreCalculus</u>

Dividing Polynomials

1. When you divide a polynomial by a binomial and there is no remainder, what does that tell you about the quotient and the divisor?

2. Explain whether or not the remainder will always be a constant when a polynomial in *x* is divided by a binomial of the form *x* – *a*.

3. When dividing a polynomial by another polynomial, suppose the remainder is not 0 but its degree is less than that of the divisor. Explain whether or not the division could continue.

4. Divide using long division. State the quotient, q(x), and the remainder, r(x).
   a. $(x^3 - 2x^2 - 5x + 6) \div (x - 3)$
   b. $\frac{3x^2-2x+5}{x-3}$
   c. $\frac{x^4-81}{x-3}$
   d. $\frac{2x^5-8x^4+2x^3+x^2}{2x^3+1}$

5. Divide using synthetic division:
   a. $(5x^3 - 6x^2 + 3x + 11) \div (x - 2)$
   b. $(x^2 + x - 2) \div (x - 1)$

6. Use synthetic division and the Remainder Theorem to find the indicated value:
   $f(x) = x^3 - 7x^2 + 5x - 6$ for $f(3)$

7. Solve the equation $2x^3 - 3x^2 - 11x + 6 = 0$ given that -2 is a zero of $f(x) = 2x^3 - 3x^2 - 11x + 6$.

8. Without actually dividing, find the remainder when $x^{39} + 1$ is divided by $x - 1$.

Selected problems were taken from Blitzer's <u>PreCalculus</u>