PRECALCULUS PROBLEM SESSION #6

Zeros of Polynomial Functions

1. Explain whether the Rational Zeros Theorem guarantees that a polynomial function with integral coefficients has any rational zeros.

2. Use the Rational Zero Theorem to list all possible rational zeros:
   a. $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$ b. $f(x) = 3x^4 - 11x^3 - 3x^2 - 6x + 8$

3. Can an *n*th-degree polynomial function have more than *n* zeros? Why or why not?

4. If *Q(x)* = -*P(x),* do *P(x)* and *Q(x)* have the same zeros? Why or why not?

5. Find all possible rational zeros. Use the synthetic division to test the possible rational zeros and find an actual zero. Then, find the remaining zeros of the polynomial function.
   a. $f(x) = 2x^3 - 5x^2 + x + 2$ b. $f(x) = x^3 - 2x^2 - 11x + 12$

6. Find all possible rational roots. Use synthetic division to test the possible rational roots and find an actual root, then find the remaining roots and solve the equation $x^4 - 2x^2 - 16x - 15 = 0$

7. Let *P(x)* be a third-degree polynomial with real coefficients. Indicate which of the following statements are true and which are false. Justify your conclusions.
   (a) *P(x)* has at least one real zero. (b) *P(x)* has three zeros.
   (c) *P(x)* can have two real zeros and one imaginary zero.

8. a) Explain why *P(x)* = $x^3 - x - 1$ has no rational zeros. b) Explain why *P*(*x*) = $x^3 + k$ has at least one real zero.

9. Explain why the graph of a cubic function must touch or cross the *x*-axis at least once.

10. Find an nth-degree polynomial function with real coefficients satisfying the given conditions. If using a graphing utility, use it to graph the function and verify the real zeros and the given function value.
    a) n = 3; 4 and $3i$ are real zeros; $f(-1) = -50$ b) n = 3; 6 and $-5 + 2i$ are zeros; $f(2) = -636$

11. Explain the difference between a polynomial function and a rational function.

Rational Functions and Their Graphs

1. Explain the difference between a polynomial function and a rational function.

2. Under what circumstances will a rational function have a domain consisting of all real numbers?

3. Explain why the graph of a rational function cannot have both a horizontal and an oblique asymptote.

4. Find the domain: $g(x) = \frac{2x^2}{(x-2)(x+6)}$

5. Find the vertical asymptote: $g(x) = \frac{x+3}{x(x-3)}$

6. Find the horizontal asymptote: $g(x) = \frac{15x^2}{3x^2+1}$

Selected problems were taken from Blitzer's PreCalculus

7. Explain whether x = 2 is a vertical asymptote for the function $f(x) = \dfrac{x^2 - 4}{x - 2}$.

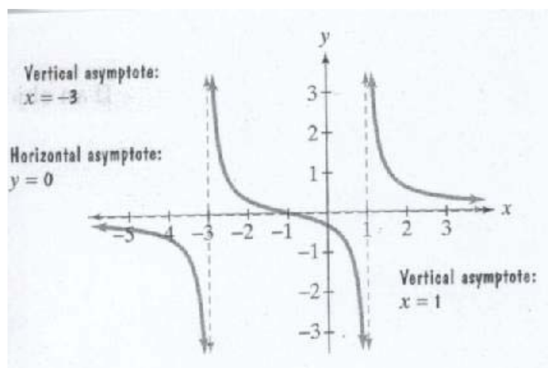
8. Use transformations of $f(x) = \frac{1}{x}$ or $f(x) = \frac{1}{x^2}$ to graph a) $g(x) = \frac{1}{x-2}$ b) $g(x) = \frac{1}{(x+1)^2}$

9. Graph: a) $f(x) = \frac{3x}{x-1}$ b) $f(x) = \frac{4x}{x^2-1}$ c) $f(x) = \frac{x-4}{x^2-x-6}$ d) $f(x) = \frac{x^2}{x^2+x-6}$

10. Graph $y_1 = \dfrac{x^3 + 4}{x}$ and $y_2 = x^2$ using the same graph. Explain how the parabola $y_2 = x^2$ can be thought of as a nonlinear asymptote for y_1.

11. Find the slant asymptotes of the graph of each rational function and follow the seven step strategy and use the slant asymptote to graph the rational function: $f(x) = \frac{x^2-x+1}{x-1}$

12. Use the graph to complete each statement:
    As $x \to -3^+$, $f(x) \to$______
    As $x \to 1^+$, $f(x) \to$______
    As $x \to \infty$, $f(x) \to$______



Polynomial and Rational Inequalities

1. Solve. Graph the solution set on a real number line, and show in interval notation.
    a) $(x + 1)(x + 2)(x + 3) \geq 0$
    b) $x^2 - 4x + 3 < 0$

2. Solve each rational inequality and graph the solution set on a real number line. Express each solution set in interval notation.
    a) $\frac{x+5}{x-2} > 0$ b) $\frac{1}{x-3} < 1$ c) $\frac{x}{x-1} > 2$

3. Why, when solving rational inequalities, do we need to find values for which the function is undefined as well as zeros of the function?

4. Find the domain of the function: $f(x) = \sqrt{\frac{x}{2x-1} - 1}$

5. Write a quadratic inequality for which the solution set is (-4, 3).

6. Write a polynomial inequality for which the solution set is [-4, 3] $\bigcup$ [7, ∞].

7. Solve: $(x - 2)^{-3} < 0$

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