PRECALCULUS PROBLEM SESSION #8

Properties of Logarithms

1. Explain the errors, if any, in the following: $\log_a ab^3 = (\log_a a)(\log_a b^3) = 3\log_a b$

2. Use the properties of logarithms to expand each logarithmic expression as much as possible.
   a. $\log\left(\frac{x}{1000}\right)$ b. $\ln\sqrt[7]{x}$ c. $\ln\left[\frac{x^4\sqrt{x^2+3}}{(x+3)^5}\right]$ d. $\log_9 9x$ e. $\log_b\left(\frac{\sqrt[3]{x}y^4}{z^5}\right)$

3. Use properties of logarithms to condense each logarithmic expression. Write the expression as a single logarithm whose coefficient is 1.
   a. $8\ln(x+9) - 4\ln x$ b. $\frac{1}{3}(\log_4 x - \log_4 y) + 2\log_4(x+1)$

4. State whether each of the following is true or false.
   a. $\frac{\log_b M}{\log_b N} = \log_b M - \log_b N$ b. $\log_b(M+N) = \log_b M + \log_b N$ c. $(\log_b M)^p = p(\log_b M)$

5. Write each of the following expressions $\log_b \frac{x^2}{\sqrt{x+1}}$ in terms of logarithms of first-degree polynomials.
   a. $\log_b \frac{(x+10)^7}{(1+10x)^2}$ b. $\log_b(x^4 + x^3 - 20x^2)$

Exponential and Logarithmic Equations

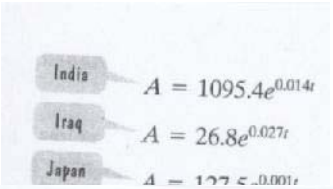
1. Solve by expressing each side as a power of the same base and the equating the exponents.
   a. $8^{1-x} = 4^{x+2}$ b. $5^{2-x} = \frac{1}{125}$

2. Solve and express the solution set in terms of natural logarithms.
   a. $5^{x-3} = 137$ b. $e^{1-8x} = 7957$

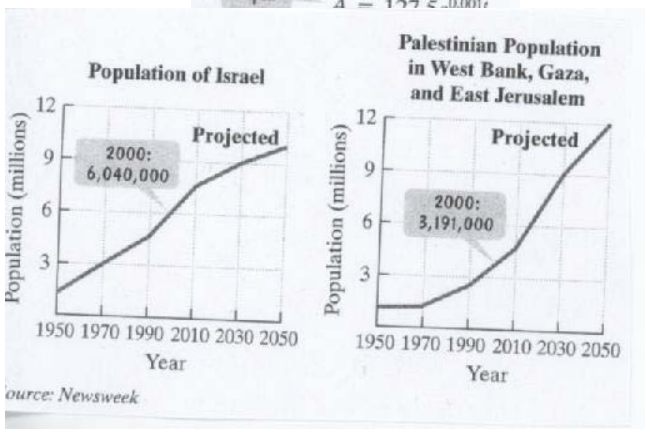
3. Solve each logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give the exact answer, and then give an approximate decimal answer.
   a. $\log_5(x-7) = 2$ b. $\log(x+7) - \log 3 = \log(7x+1)$ c. $\log_6(x+5) + \log_6 x = 2$
   d. $\log(5x+1) = \log(2x+3) + \log 2$

Exponential Growth and Decay

1. The exponential models describe the population of the indicated country, A, in millions, t, after 2006. Use the models to answer the following questions.
   a. What was the population of Iraq in 2006?
   b. When will India's population be 1416 million?



2. The graphs show that by 2050, Palestinians in the West Bank, Gaza Strip, and East Jerusalem will outnumber Israelis. Use the graph to help answer the following.
   a. In 2000, the population of the Palestinians in West Bank, Gaza Strip, and East Jerusalem was approximately 3.2 million and by 2050 it is projected to grow to 12 million. Use the exponential growth model $A = A_0 e^{kt}$, in the



Selected problems were taken from Blitzer's PreCalculus

which t is the number of years after 2000, to find the exponential growth function that models the data.

b. In which year will the Palestinian population be 9 million?

3. An artifact originally had 16 grams of carbon-14 present. The decay model $A = 16e^{-0.000121t}$ describes the amount of carbon-14 present after t years. How many grams of carbon-14 will be present in 11,430 years?

4. The half-life of the radioactive element plutonium-230 is 25,000 years. If 16 grams of plutonium are initially present, how many grams are present after 25,000 years? 50,000 years? 75,000 years? 100,000 years? 125,000 years?

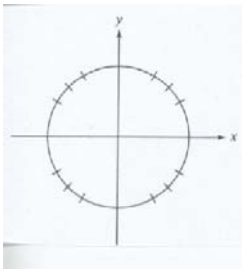
5. Use the exponential decay model for carbon-14, $A = 16e^{-0.000121t}$ to solve. Skeletons were found at a construction site in San Francisco in 1989. The skeletons contained 88% of the expected amount of carbon-14 found in a living person. In 1989, how old were the skeletons?

6. Use the exponential decay model, $A = A_0e^{kt}$ to solve. The half-life of aspirin in your bloodstream is 12 hours. How long will it take for the aspirin to decay to 70% of the original dosage?

7. The growth model, $A = 107.4e^{0.012t}$ describes Mexico's population, A, is millions, t years after 2006. What is Mexico's growth rate? How long will it take Mexico to double its population?

Angles and Radian Measure

1. Explain what is meant by the statement "230° and $\frac{5\pi}{4}$ radians are third-quadrant angles."
2. Give the measures of any two angles coterminal with the given angles: a. 130° b. 3π radians
3. Convert each angle in degrees to radians: a. -270° b. 150°
4. Convert each angle in radians to degrees: a. $\frac{3\pi}{4}$ b. -4π
5. Draw each angle in standard position.
   a. $\frac{4\pi}{3}$ b. $-\frac{5\pi}{6}$ c. 150°



6. Find a positive angle less than 360° or 2π that is conterminal with: $\frac{25\pi}{6}$ b. $-\frac{38\pi}{9}$
7. Find the radian measure of the central angle of a circle with a radius of 5 feet that intercepts and arc of 30 feet.
8. The minute hand of a clock moves from 12 to 4 o'clock, or $\frac{1}{3}$ of a complete revolution. Through how many degrees does it move? Through how many radians does it move?
9. The minute hand of a clock is 6 inches long and moves from 12 to 4 o'clock. How far does the tip of the minute hand move? Express your answer in terms of π and then round to two decimal places.
10. A Ferris wheel has a radius of 25 feet. The wheel is rotating at two revolutions per minute. Find the linear speed, in feet per minute, of the water.

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