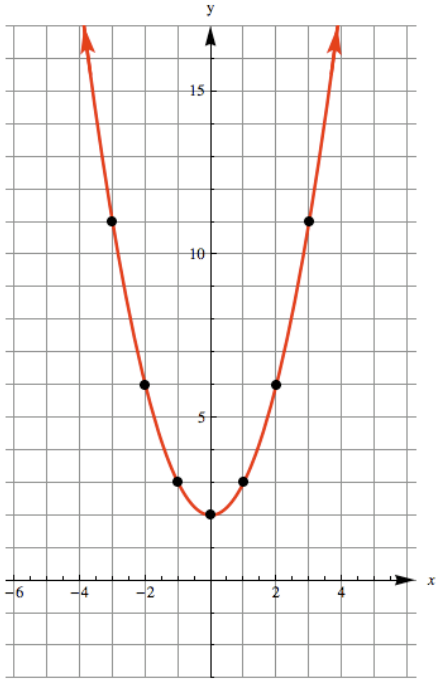
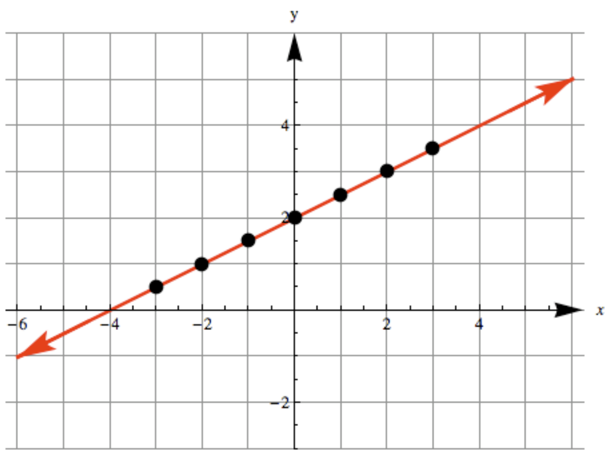
PRECALCULUS PROBLEM SESSION # 1 SOLUTIONS

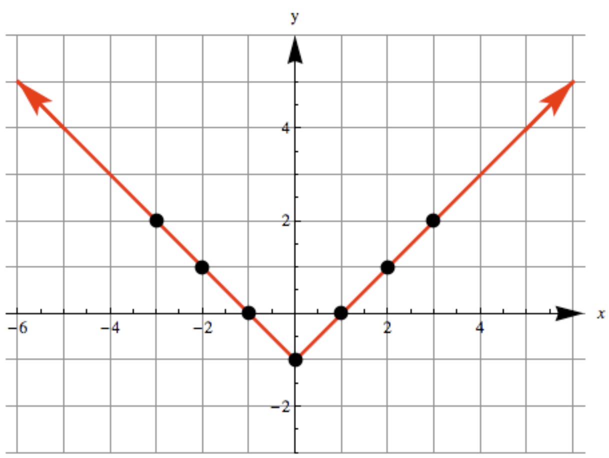
Functions and Their Graphs

1. a) 

x = -3, y = 11
x = -2, y = 6
x = -1, y = 3
x = 0, y = 2
x = 1, y = 3
x = 2, y = 6
x = 3, y = 11

b) 

x = -3, y = 1/2
x = -2, y = 1
x = -1, y = 3/2
x = 0, y = 2
x = 1, y = 5/2
x = 2, y = 3
x = 3, y = 7/2

c) 

x = -3, y = 2
x = -2, y = 1
x = -1, y = 0
x = 0, y = -1
x = 1, y = 0
x = 2, y = 1
x = 3, y = 2

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2. a) -1, 1; There are 2 x-intercepts located at (-1, 0) and (1, 0).
   b) 1; There is 1 y-intercept at (0, 1).

Introduction to Functions

1. Domain of a function is the set of all possible input values of the function, while the range of a function is the set of all possible output values of the function.

   a) The relation is not a function since the two ordered pairs (5, 6) and (5, 7) have the same first component but different second components (the same can be said for the ordered pairs (6, 6) and (6, 7)). The domain is {5, 6} and the range is {6, 7}.

   b) The relation is a function because no two ordered pairs have the same first component and different second components. The domain is {−2, −1, 5, 10} and the range is {1, 4, 6}.

2. a) The vertical line test fails so y is not a function of x.

   b) The vertical line test fails so y is not a function of x.

3. a) $g(-1) = 8$ b) $g(x + 2) = x^2 - 6x - 19$ c) $g(-x) = x^2 + 10x - 3$

4. A linear function $f(x) = mx + b$ with $m \neq 0$ has exactly one zero at $(-b/m, 0)$.

5. Yes, it is true that for some functions $f(a + b) = f(a) + f(b)$.
   For example, $f(x) = x^2 + x - 7$. Then we have the following:
   $f(a+b) = 4(a+b)^2 + (a+b) - 7 = 4a^2 + 4b^2 + 8ab + a + b - 7$ and
   $f(a) + f(b) = (4a^2 + a - 7) + (4b^2 + b - 7) = 4a^2 + 4b^2 + a + b - 14$, setting the two equal to each other we get:
   $4a^2 + 4b^2 + 8ab + a + b - 7 = 4a^2 + 4b^2 + a + b - 14 \Rightarrow 8ab - 7 = -14 \Rightarrow ab = -\frac{7}{8} \Rightarrow a = -\frac{7}{8b}$ Now choosing any value for b, we can find a corresponding value a which makes the statement $f(a+b) = f(a) + f(b)$ true, for example, try $b = 1$ and $a = -\frac{7}{8}$.

6. a) Solving the equation for y gives $y = 25 - x^2$. Since only one value of y can be obtained for each value of x, y is a function of x.

   b) Solving the equation for y gives $y = \pm\sqrt{25 - x^2}$. If $x = 0$, $y = \pm 5$. Since two values, $y = -5$ and $y = 5$, can be obtained for one value of x, y is not a function of x.

   c) Solving the equation for y gives $y = \sqrt[3]{27 - x}$. Since only one value of y can be obtained for each value of x, y is a function of x.

7. $V(x) = 22{,}500 - 3{,}200x$, $0 \leq x \leq 6$ ($).
   $V(3)$ = $12,900, thus after 3 years, the car will be worth $12,900.

8. a) domain: (−6, 0]
   b) range: [−3, 4)

c) x-intercept: (–3.75, 0)
d) y-intercept: (0, –3)

More on Functions

1. If a vertical line, say $x = a$, intersects a graph in more than one point then there exists more than one output value for the input value a, which implies that the relation represented by the graph is not a function.

2. Left graph:
a. increasing: (–5, –4) and (–2, 0) and (2, 4)
b. decreasing: (–4, –2) and (0, 2) and (4, 5)
c. constant: none

Middle graph:
a. increasing: (–∞, -1)
b. decreasing: (–1, ∞)
c. constant: none

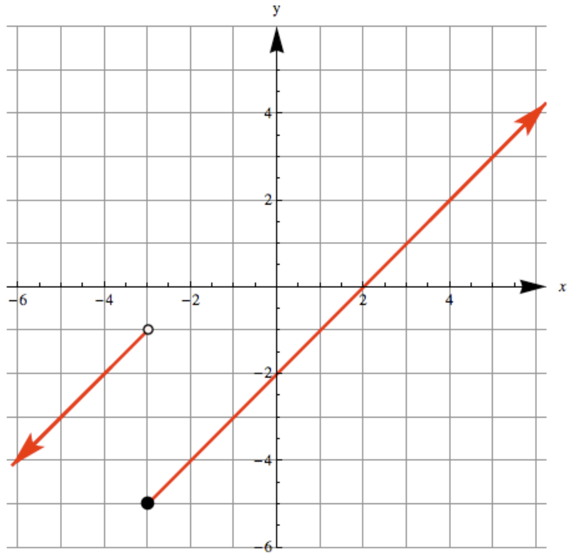
Right graph:
a. increasing: (0, ∞)
b. decreasing: none
c. constant: (–∞, 0)

3. Yes, it is has all three symmetries; it is both even and odd; it is the only function with such properties, because for any other function, x-axis symmetry implies that the vertical line test fails.

4. a) neither an odd function nor an even function
b) odd function
c) even function

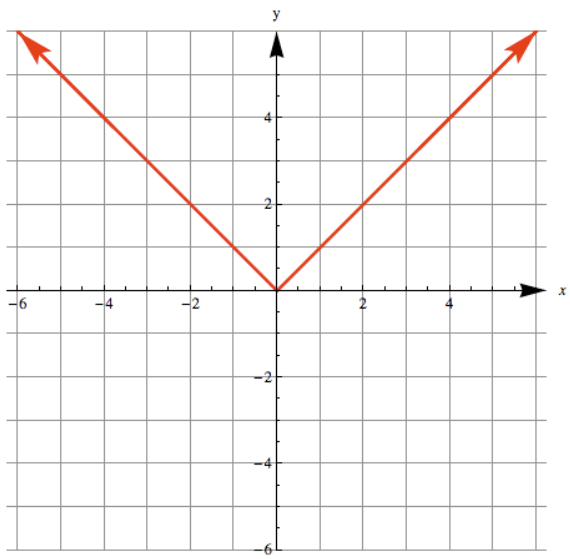
5. The left graph is not symmetric with respect to the y-axis or the origin. The function is neither even nor odd.
The right graph is symmetric with respect to the origin. The function is an odd function.

6. a)



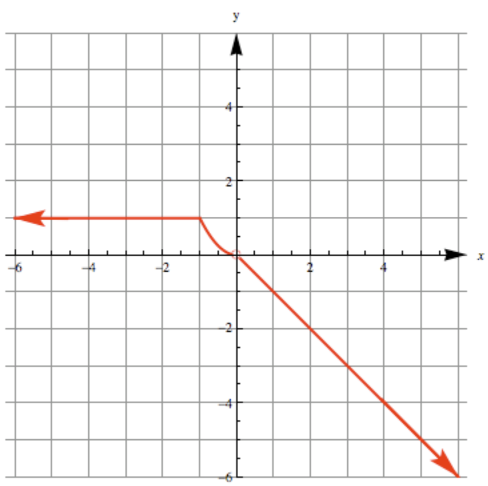
Range: (−∞, ∞)

b)



Range: (0, −∞]

c)



Range: (−∞, 0) ∪ (0, 1]

7. a) $-2x-h-3$

   b) $-\frac{1}{2x(x+h)}$

8. a) domain: $(-\infty, \infty)$
   b) range: $(-\infty, 4]$
   c) x-intercepts: $(-4, 0)$, $(4, 0)$
   d) y-intercept: $(0, 1)$
   e) increasing: $(-\infty, -2)$ and $(0, 3)$
   f) decreasing: $(-2, 0)$ and $(3, \infty)$
   g) $(-\infty, -4]$ and $[4, \infty)$
   h) $x = -2$ and $x = 3$
   i) $f(-2) = 4$ and $f(3) = 2$
   j) $x = 0$
   k) $f(0) = 1$
   l) $f(-2) = 4$
   m) $x = -4$ and $x = 4$
   n) neither; $f(-x) \neq x$, $f(-x) \neq -x$

9. One such example could be the temperature of water in a pot. When bringing the water to a boiling temperature (increasing), then keep it boiling for a bit (constant), and, then, letting it cool down (decreasing).