PRECALCULUS PROBLEM SESSION #11 SOLUTIONS

Inverse Trigonometric Functions

1. a) $\pi/3$ b) $-\pi/3$ c) 0 d) $-\pi/6$
2. a) $2\pi/3$ b) $2\pi/3$ c) $-\pi/4$
3. Because the range of $\sin^{-1}(x)$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
4. a) $12/5$ b) $\frac{2\sqrt{3}}{3}$
5. (a) $\sqrt{1-4x^2}$ (b) x
6. The ranges of some inverse trigonometric functions are restricted: $\sin^{-1} x$, $\cos^{-1} x$, $\csc^{-1} x$ and $\sec^{-1} x$. This is because the domains of the corresponding trigonometric functions are restricted, in order for them to become one-to-one functions, so that they may have inverses, and since the domain of a function becomes the range of the inverse, it follows that the ranges of these inverse functions are restricted.

Applications of Trigonometric Functions

1. C = 90°, B = 48.5°, a = 17.6945, c = 26.7038
2. $-8\cos(\pi t)$
3. a) 8 inches b) ¼ cycle per second c) The time required for one cycle is 4 seconds.
4. The height of the building is approximately 24 feet.
5. The stolen car is approximately 260 feet from a point directly below the helicopter.
6. The angle of elevation of the sun is approximately 80.9°.
7. The height of the flagpole is approximately 209.8 feet.

Verifying Trigonometric Identities

1. $\cos^2 x + \sin^2 x = 1$

$$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \Rightarrow 1 + \tan^2 x = \sec^2 x$$

$$\frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} \Rightarrow \cot^2 x + 1 = \csc^2 x$$

2. A trigonometric equation that is an identity is ALWAYS true, while a trigonometric equation that is not an identity is only true for certain values of the variable.
$\cos^2 x + \sin^2 x = 1$ is always true, while $\cos x$ = ½ is sometimes true.

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3. a)
$$\cos x \csc x = \cos x \cdot \frac{1}{\sin x} = \frac{\cos x}{\sin x} = \cot x$$

b)
$$\csc x - \csc x \cos^2 x = \csc x(1-\cos^2 x) = \frac{1}{\sin x} \cdot \sin x^2 = \sin x$$

c)
$$\frac{\cos\theta \sec\theta}{\cot\theta} = \frac{\frac{\cos\theta}{1} \cdot \frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}} = \frac{1}{\frac{\cos\theta}{\sin\theta}} = 1 \div \frac{\cos\theta}{\sin\theta} = 1 \cdot \frac{\sin\theta}{\cos\theta} = \tan\theta$$

d)
$$\frac{1-\sin^2 t}{\sin t} = \frac{\cos^2 t}{\sin t} = \cos t \cdot \frac{\cos t}{\sin t} = \cos t \cot t$$

e)
$$\frac{\cot^2 t}{\csc t} = \frac{\csc^2 t - 1}{\csc t} = \frac{\csc^2 t}{\csc t} - \frac{1}{\csc t} = \csc t - \sin t$$

f)
$$\frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \sec\theta - \tan\theta$$

g)
$$1 - \frac{\cos^2 x}{1+\sin x} = 1 - \frac{\cos^2}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x} = 1 - \frac{\cos^2 x(1-\sin x)}{1-\sin^2 x} = 1 - \frac{\cos^2 x(1-\sin x)}{\cos^2 x} = 1 - 1 + \sin x = \sin x$$

h)
$$\frac{\sin x}{\cos x+1} + \frac{\cos x - 1}{\sin x} = \frac{\sin x}{\cos x+1} \cdot \frac{\cos x-1}{\cos x-1} + \frac{\cos x-1}{\sin x} = \frac{\sin x(\cos x-1)}{\cos^2 x - 1} + \frac{\cos x-1}{\sin x} = \frac{\sin x(\cos x-1)}{-\sin^2 x} + \frac{\cos x-1}{\sin x} = \frac{\sin x(1-\cos x)}{\sin^2 x} + \frac{\cos x-1}{\sin x} = \frac{1-\cos x}{\sin x} + \frac{\cos x-1}{\sin x} = \frac{0}{\sin x} = 0$$

i)
$$\frac{\csc x - \sec x}{\csc x + \sec x} = \frac{\frac{1}{\sin x} - \frac{1}{\cos x}}{\frac{1}{\sin x} + \frac{1}{\cos x}} = \frac{\frac{1}{\sin x} - \frac{1}{\cos x}}{\frac{1}{\sin x} + \frac{1}{\cos x}} \cdot \frac{\cos x}{\cos x} = \frac{\frac{\cos x}{\sin x} - 1}{\frac{\cos x}{\sin x} + 1} = \frac{\cot x - 1}{\cot x + 1}$$

j)
$$\frac{\tan^2 x - \cot^2 x}{\tan x + \cot x} = \frac{(\tan x - \cot x)(\tan x + \cot x)}{\tan x + \cot x} = \tan x - \cot x$$

k)
Left side:
$$\frac{\cos^2 t + 4\cos t + 4}{\cos t + 2} = \frac{(\cos t + 2)(\cos t + 2)}{\cos t + 2} = \cos t + 2$$
Right side:
$$\frac{2\sec t + 1}{\sec t} = \frac{2\sec t}{\sec t} + \frac{1}{\sec t} = 2 + \cos t = \cos t + 2$$
The identity is verified because both sides are equal to $\cos t + 2$.

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Sum and Distance Formulas

1. a) $\frac{\sqrt{2}+\sqrt{6}}{4}$ b) $\frac{\sqrt{2}-\sqrt{6}}{4}$ c) $-2-\sqrt{3}$

2. a) $\frac{\sqrt{3}}{3}$ b) 1

3.
$$\begin{aligned}&\cos(\alpha+\beta)+\cos(\alpha-\beta)\\&=\cos\alpha\cos\beta-\sin\alpha\sin\beta\\&+\cos\alpha\cos\beta+\sin\alpha\sin\beta\\&=2\cos\alpha\cos\beta\end{aligned}$$

4. **Part a)**

   a) –4/5 b) –3/5 c) 3/4

   Part b)

   a) $\frac{-6-4\sqrt{5}}{15}$ b) $\frac{8-3\sqrt{5}}{15}$ c) $\frac{54-25\sqrt{5}}{22}$