PRECALCULUS PROBLEM SESSION #12 SOLUTIONS

Double-Angle, Power-Reducing, and Half-Angle Formulas

1. **Part i)**
   a) $-\frac{120}{169}$ b) $-\frac{119}{169}$ c) $\frac{120}{169}$
   Part ii)
   a) $\frac{3}{5}$ b) $\frac{4}{5}$ c) $\frac{3}{4}$
2. a) $\frac{\sqrt{2+\sqrt{2}}}{2}$ b) $\frac{\sqrt{2+\sqrt{3}}}{2}$ c) $\sqrt{2}+1$
3. Solving for (a) $\sin\frac{\alpha}{2}$, (b) $\cos\frac{\alpha}{2}$, and (c) $\tan\frac{\alpha}{2}$
   Part a
   (a) $\frac{4\sqrt{17}}{17}$ (b) $-\frac{\sqrt{17}}{17}$ (c) -4
   Part b
   (a) $\frac{\sqrt{6}}{3}$ (b) $\frac{\sqrt{3}}{3}$ (c) $\sqrt{2}$
4. $\frac{15}{4}+5\cos 2x+\frac{5}{4}\cos 4x$
5. Errors: $\mathbf{\cos 4x \neq 2\cos 2x}$ and $\mathbf{\cos 2x \neq (\cos^2 x+\sin^2 x)}$

   Verifying with $x=\pi$ gives:
   $$2\sin^2 2\pi+\cos 4\pi=1$$
   $$8\sin^2\pi\cos^2\pi+2=2$$
   $$1\neq 2$$
6. $\cos 2A = \cos^2 A - \sin^2 A = = \cos^2 A-\sin^2 A+\cos^2 A-\cos^2 A = 2\cos^2 A-(\sin^2 A+\cos^2 A) =$

   $= 2\cos^2 A-1$

   $\cos 2A = \cos^2 A - \sin^2 A = = \cos^2 A-\sin^2 A+\sin^2 A-\sin^2 A = (\cos^2 A+\sin^2 A)-2\sin^2 A =$

   $=1-2\sin^2 A$

Trigonometric Equations

1. (A) The equation has two solutions on the interval $[0, 2\pi)$: $x=\frac{\pi}{3}$ and $x=\frac{5\pi}{3}$.

   (B) The equation has infinitely many solutions on the interval $(-\infty, \infty)$:

   $x=\frac{\pi}{3}+2k\pi$, k is any integer

   $x=\frac{5\pi}{3}+2k\pi$, k is any integer

2. a) $x=\frac{\pi}{6}+2k\pi$ or $x=\frac{11\pi}{6}+2k\pi$, k is any integer

b) $x = \frac{\pi}{3} + k\pi$, k is any integer

c) $x = \frac{4\pi}{3} + 2k\pi$ or $x = \frac{5\pi}{3} + 2k\pi$, k is any integer

d) $x = \frac{\pi}{8} + k\pi$ or $x = \frac{15\pi}{8} + k\pi$, k is any integer

3. a) The solutions are $\frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, and\ \frac{15\pi}{8}$.

   b) The solution is $\frac{\pi}{3}$.

   c) The solutions are $\frac{\pi}{6}, \frac{5\pi}{6}, and\ \frac{3\pi}{2}$.

   d) The solution is 0.

   e) The solutions are $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, and\ \frac{5\pi}{3}$.

4. No, the equation $5 \sin x = 7$ does not have a solution, since the function $y = 5 \sin x$ has an amplitude of 5 and therefore will never be equal to 7.

5. No, Jan's and Jacob's answers are different. $\frac{\pi}{6} + k\pi = \frac{7\pi}{6} + (2\pi)k$ only when k is odd. Thus the expression $\frac{\pi}{6} + k\pi$ has twice as many values as the expression $\frac{7\pi}{6} + (2\pi)k$ does.

6. The solutions are $\frac{\pi}{6}, \frac{5\pi}{6}, and\ \frac{3\pi}{2}$.