PRECALCULUS PROBLEM SESSION #3 SOLUTIONS

Inverse Functions

1.

$$f(x) = 4x + 9;\ g(x) = \frac{x-9}{4}$$

$$f(g(x)) = 4\left(\frac{x-9}{4}\right) + 9 = x - 9 + 9 = x$$

$$g(f(x)) = \frac{(4x+9)-9}{4} = \frac{4x}{4} = x$$

f and g are inverses.

2. a)

$$f(x) = x^3 - 1$$
$$y = x^3 - 1$$

$$x = y^3 - 1$$
$$x + 1 = y^3$$
$$y = \sqrt[3]{x+1}$$
$$f^{-1}(x) = \sqrt[3]{x+1}$$
$$f\left(f^{-1}(x)\right) = (\sqrt[3]{x+1})^3 - 1$$
$$= x + 1 - 1$$
$$= x$$
$$f^{-1}\left(f(x)\right) = \sqrt[3]{x^3 - 1 + 1} = \sqrt[3]{x^3 = x}$$

2. b)

$$f(x) = \frac{4}{x} + 9$$
$$y = \frac{4}{x} + 9$$

$$x = \frac{4}{y} + 9$$
$$xy = 4 + 9y$$
$$y(x - 9) = 4$$
$$y = \frac{4}{x-9}$$
$$f^{-1}(x) = \frac{4}{x-9}, \quad x \neq 9$$
$$f(f^{-1}(x)) = \frac{4}{\frac{4}{x-9}} + 9 = x$$
$$f^{-1}(f(x)) = \frac{4}{\frac{4}{x+9-9}} = x$$

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2. c)

$$f(x) = \frac{2x-3}{x+1}$$
$$y = \frac{2x-3}{x+1}$$
$$x = \frac{2y-3}{y+1}$$
$$xy + x = 2y - 3$$
$$y(x-2) = -x - 3$$
$$y = \frac{-x-3}{x-2}$$

$$f^{-1}(x) = \frac{-x-3}{x-2},\ x \neq 2$$

$$f\left(f^{-1}(x)\right) = \frac{2\left(\frac{-x-3}{x-2}\right)-3}{\frac{-x-3}{x-2}+1}$$

$$= \frac{-2x-6-3x+6}{-x-3+x-2} = \frac{-5x}{-5} = x$$

$$f^{-1}\left(f(x)\right) = \frac{-\left(\frac{2x-3}{x+1}\right)-3}{\frac{2x-3}{x+1}-2}$$

$$= \frac{-2x+3-3x-3}{2x-3-2x-2} = \frac{-5x}{5} = x$$

2. d)

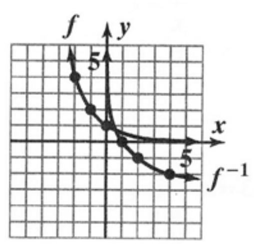
$$f(x) = 3x - 1$$
$$y = 3x - 1$$
$$x = 3y - 1$$
$$x + 1 = 3y$$
$$y = \frac{x+1}{3}$$
$$f^{-1}(x) = \frac{x+1}{3}$$

$$f(f^{-1}(x)) = 3\left(\frac{x+1}{3}\right) - 1 = x + 1 - 1 = x$$

$$f^{-1}(f(x)) = \frac{3x-1+1}{3} = \frac{3x}{3} = x$$

3. a) The function passes the Horizontal Line Test (HLT), so it does have an inverse function.
   b) The function fails the Horizontal Line Test (HLT), so it does not have an inverse function.

4.



$$f(x) = x^3 - 1$$
$$y = x^3 - 1$$
$$x = y^3 - 1$$
$$x + 1 = y^3$$
$$y = \sqrt[3]{x+1}$$
$$f^{-1}(x) = \sqrt[3]{x+1}$$

$$f(f^{-1}(x)) = \left(\sqrt[3]{x+1}\right)^3 - 1$$
$$= x + 1 - 1$$
$$= x$$

$$f^{-1}(f(x)) = \sqrt[3]{x^3 - 1 + 1} = \sqrt[3]{x^3} = x$$

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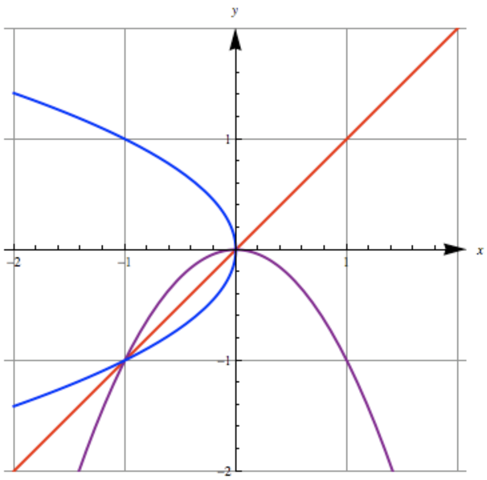
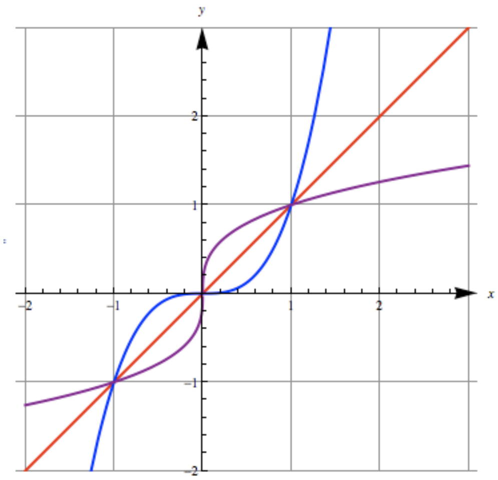
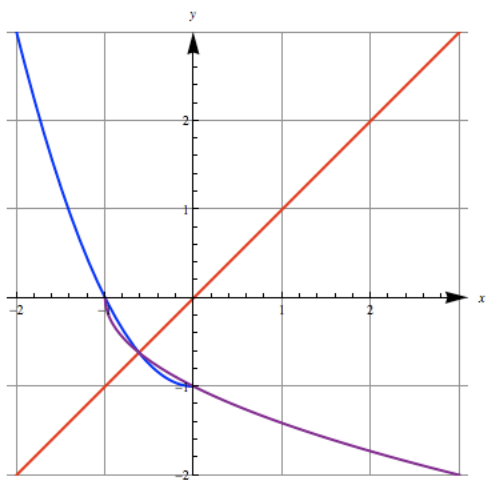
5. a)  b) 

6. $f^{-1}(x) = -\sqrt{x}$ 

domain of f: $(-\infty, 0]$
range of f: $[-1, \infty)$

domain of f^{-1}: $[-1, \infty)$
range of f^{-1}: $(-\infty, 0]$

7. Restrict the domain of f to $x \geq 0$, then $(f^{-1})(x) = \sqrt{x+3}$ and the domain of f^{-1} is $x \geq -3$.

8. The graph of $f(x)$ is reflected over the line $y = x$.

9. $(f^{-1})(x) = \dfrac{x-b}{m}$, so $f(f^{-1}(a)) = f(\dfrac{a-b}{m}) = m\dfrac{(a-b)}{m} + b = a$

10. Slope: $\dfrac{1}{m}$; y-intercept: $(0, -\dfrac{b}{m})$

11. Yes; one example is the "sideways" parabola defined by $x = y^2$ which is not a function, but its inverse $y = x^2$ is a function.

12. $f^{-1}(x)$ is the inverse function of $f(x)$, while $[f(x)]^{-1} = \dfrac{1}{f(x)}$, that is, the reciprocal of $f(x)$.

Distance and Midpoint Formulas and Circles

1. $\left(-\dfrac{2}{5}, \dfrac{1}{10}\right)$

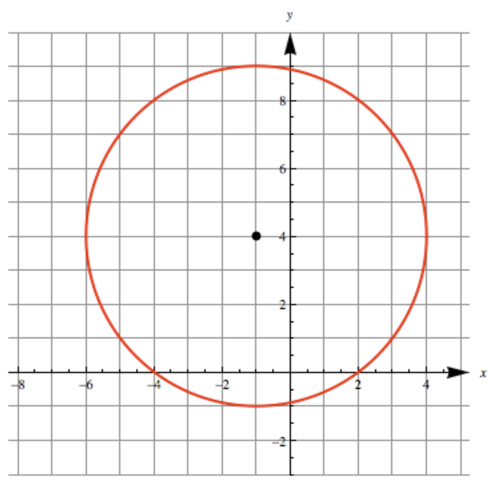
PRECALCULUS PROBLEM SESSION #3 SOLUTIONS

2. It is the line perpendicular to the line segment joining (1, 1) and (3, 0) and passing through the midpoint of the segment, that is, the line $y = 2x - \frac{7}{2}$.

   Using the distance formula: $dist((1,1),(x,y)) = dist((3,0),(x,y))$

$$\sqrt{(1-x)^2 + (1-y)^2} = \sqrt{(3-x)^2 + (0-y)^2}$$
$$(1-x)^2 + (1-y)^2 = (3-x)^2 + (0-y)^2$$
$$1 - 2x + x^2 + 1 - 2y + y^2 = 9 - 6x + x^2 + y^2$$
$$-2y = 7 - 4x$$
$$y = 2x - \frac{7}{2}$$

3. $\sqrt{17}$

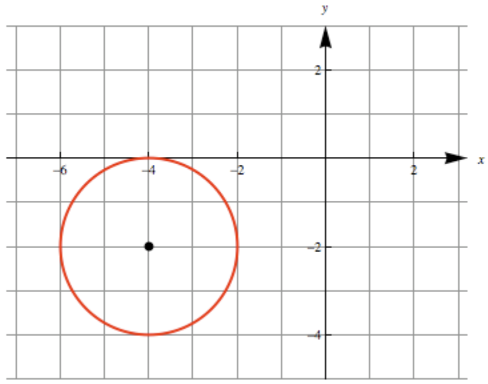
4. 

Center = $(h, k) = (-1, 4)$

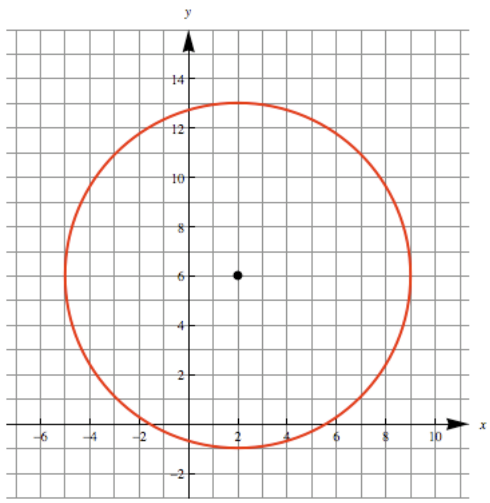
Radius = $r = 5$

Domain = $[-6, 4]$

Range = $[-1, 9]$

5. a) 

Center = $(h, k) = (-4, -2)$, Radius = $r = 2$

b) 

Center = $(h, k) = (2, 6)$, Radius = $r = 7$

Real World Problems

1. a) $f(x) = 180 + 0.25x$ b) You drove 860 miles for $395.

2. a) $V(x) = 4x^3 - 120x^2 + 900x$
   b) If 3 inches are cut from each side, the volume will be 1,728 square inches.
   If 4 inches are cut from each side, the volume will be 1,936 square inches.
   If 5 inches are cut from each side, the volume will be 2,000 square inches.
   If 6 inches are cut from each side, the volume will be 1,944 square inches.
   If 7 inches are cut from each side, the volume will be 1,792 square inches.
   c) The volume of the box increases from 1,728 in^2 to 2,000 in^2, as the length of the side of the square cut from each corner increases from 3 inches to 5 inches. Then decreases from 2,000 in^2 to 1,792 in^2, as the side of the square cut from each corner increases from 5 inches to 7 inches.
   d) Since x is the number of inches to be cut from each side, $x > 0$. Since each side is 30 inches, you must cut less than 15 inches from each side. $0 < x < 15$ or $(0, 15)$

3. $A(x) = -2x^2 + 600x$

4. $A(x) = x^2 + \frac{2{,}000}{x}$

5. $d(x) = \sqrt{x^2 - 3x + 4}$

6. $d\,(x) = \sqrt{x^4 - 15x^2 + 64}$