PRECALCULUS PROBLEM SESSION #4 SOLUTIONS

Complex Numbers

1. a) $3 + 6i$ b) 1 c) $2 + 16i$ d) 53 e) $-\frac{3}{8} + \frac{\sqrt{7}}{16}i$ f) $-4 - 28i$ g) $-\frac{12}{13} - \frac{18}{13}i$

2. a) $-\frac{5}{13} + \frac{14}{13}i$ b) $\frac{17}{25} - \frac{6}{25}i$

3. a) The solution set is $\left\{-\frac{1}{2} + \frac{\sqrt{5}}{2}i,\ -\frac{1}{2} - \frac{\sqrt{5}}{2}i\right\}$.

   b) The solution set is $\left\{\frac{2}{3} + \frac{\sqrt{14}}{3}i,\ \frac{2}{3} - \frac{\sqrt{14}}{3}i\right\}$.

4. When $b^2 - 4ac < 0$.

Quadratic Functions

1. The vertex of the parabola is the lowest or the highest point on the parabola, therefore, its *y*-coordinate is the minimum or the maximum value the function can produce.

2. (a) and (c)

3. (a)

$f(x) = 6 - 4x + x^2$

$f(x) = x^2 - 4x + 6$

$f(x) = (x^2 - 4x + 4) + 6 - 4$

$f(x) = (x-2)^2 + 2$

vertex: (2, 2)

x-intercepts:

$0 = (x-2)^2 + 2$

$(x-2)^2 = -2$

$x - 2 = \pm i\sqrt{2}$

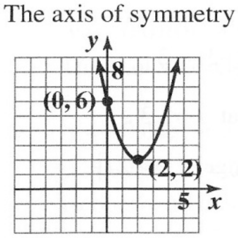
$x = 2 \pm i\sqrt{2}$

No *x*-intercepts

y-intercept:

$f(0) = 6 - 4(0) + (0)^2 = 6$

The axis of symmetry is $x = 2$.



$f(x) = 6 - 4x + x^2$

domain: $(-\infty, \infty)$

range: $[2, \infty)$

(b)

$f(x) = (x-1)^2 - 2$

vertex: (1, –2)

x-intercepts:

$0 = (x-1)^2 - 2$

$(x-1)^2 = 2$

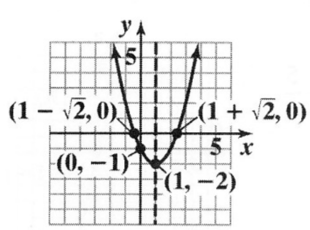
$x - 1 = \pm\sqrt{2}$

$x = 1 \pm \sqrt{2}$

y-intercept:

$f(0) = (0-1)^2 - 2 = -1$

The axis of symmetry is $x = 1$.



$f(x) = (x-1)^2 - 2$

domain: $(-\infty, \infty)$

range: $[-2, \infty)$

(c)

$f(x) = x^2 - 2x - 15$

$f(x) = (x^2 - 2x + 1) - 15 - 1$

$f(x) = (x-1)^2 - 16$

vertex: (1, –16)

x-intercepts:

$0 = (x-1)^2 - 16$

$(x-1)^2 = 16$

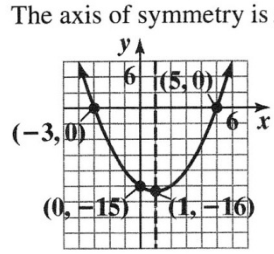
$x - 1 = \pm 4$

$x = -3$ or $x = 5$

y-intercept:

$f(0) = 0^2 - 2(0) - 15 = -15$

The axis of symmetry is $x = 1$.



$f(x) = x^2 - 2x - 15$

domain: $(-\infty, \infty)$

range: $[-16, \infty)$

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4. a) $a = -2$. The parabola opens downward and has a maximum value.
   b) The maximum is 21 at $x = -3$.
   c) Domain = $(-\infty, \infty)$, Range = $(-\infty, 21]$

5. Because if $a = 0$ then the function is linear.

6. Their graphs are symmetric with respect to the y-axis, have the same orientation, that is, they both either open downward or upward, and they have exactly the same shape (one is just a translation of the other).

7. a) The two numbers whose difference is 24 and whose product is minimized are 12 and –12. The minimum product is –144.

   b) The two numbers whose sum is 20 and whose product is maximized are 10 and 10. The maximum product is 100.

8. a) $y = -(x+2)^2 + 3$

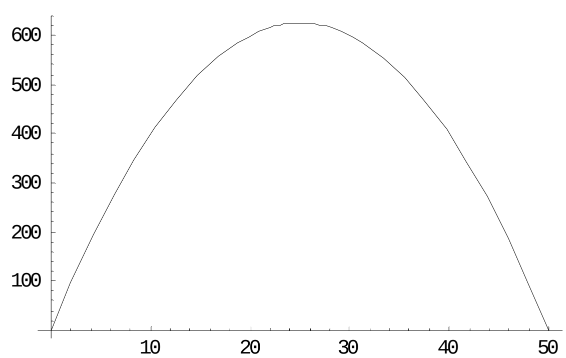
   b) $y = (x+1)^2 - 1$

9. a) The maximum height reached by the baseball after 2 seconds is 67 feet.
   b) The baseball reaches the ground after approximately 4.05 seconds.

10. The dimensions of the rectangular region with maximum area is 20 yards by 20 yards. This gives an area of 20 yards by 20 yards. This gives an area of 20 • 20 = 400 square yards.

11. (a) $A(x) = (50 - x)x = 50x - x^2$

    (b) $0 < x < 50$

    (c) 

    (d) Maximum area achieved when the dimensions are 25 feet by 25 feet

    (e) $B(x) = (100 - 2x)x = 100x - 2x^2, \quad 0 < x < 50$

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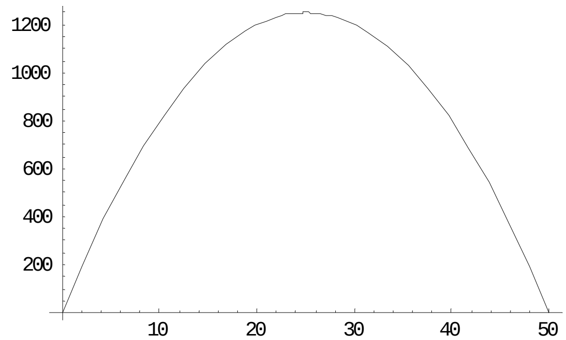

Maximum area achieved when the dimensions are 25 feet by 50 feet

12. The dimensions of the playground that maximize the total enclosed area is $\frac{200}{3} \times 100$