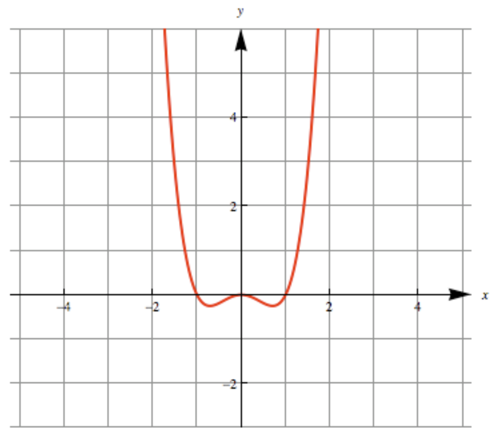
PRECALCULUS PROBLEM SESSION #5 SOLUTIONS

Polynomial Functions and Their Graphs

1. a) Yes, degree 2 b) No c) Yes, degree 1

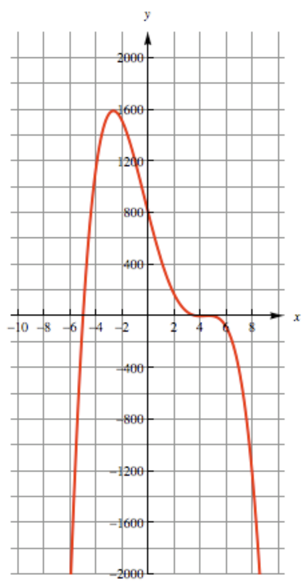
d) No e) Yes, degree 4 f) No

2. a)



Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.
x-intercepts = $(a, 0)$ = (0, 0), (-1, 0), (1, 0)
y-intercept = $(0, b)$ = (0, 0)
The graph has y-axis symmetry.

b)



Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.
x-intercepts: {(−5, 0), (4, 0), (5, 0)}
y-intercept = $(0, b)$ = (0, 800)
The graph has neither y-axis symmetry nor origin symmetry.

3. a) Approaches infinity, resembles $y = 16x^3$
b) Approaches infinity, resembles $y = 4x^4$
c) Approaches infinity, resembles $y = 2x^9$
d) Approaches infinity, resembles $y = 5x^{\frac{1}{2}} = 5\sqrt{x}$

4. a) $f(2) = -8$ and $f(3) = 81$. Thus this sign change in $f(x)$ shows there is a zero between the given values by the Intermediated Value Theorem (IVT).

b) $f(0) = 2$ and $f(1) = -1$. Thus this sign change in $f(x)$ shows there is a zero between the given values by the Intermediated Value Theorem (IVT).

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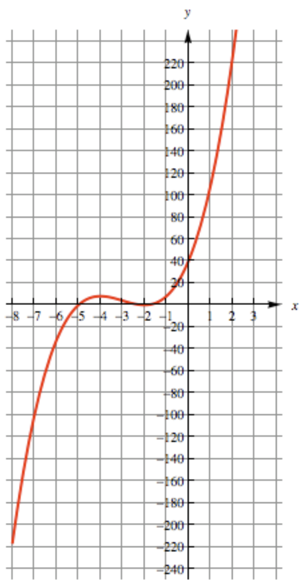
5. Zeros: $x = -5$ has multiplicity 1; the graph crosses the x-axis at $(-5, 0)$.
$x = -2$ has multiplicity 2; the graph touches the x-axis and turns around at $(-2, 0)$.

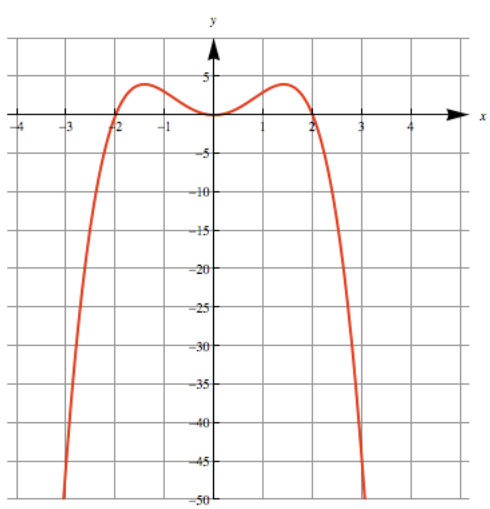


Since $a_n > 0$ and n is odd, $f(x)$ falls to the left and rises to the right.
x-intercepts: $\{(-5, 0), (-2, 0)\}$
y-intercept = $(0, b) = (0, 40)$
The graph has neither y-axis symmetry nor origin symmetry.
Some other points on the graph of $f(x)$ are:
$(-8, -216)$, $(-7, -100)$, $(-6, -32)$, $(-5, 0)$,
$(-4, 8)$, $(-3, 4)$, $(-2, 0)$, $(-1, 8)$, $(0, 40)$,
$(1, 108)$, $(2, 224)$, $(3, 400)$
The maximum number of turning points is 2.

6. The graph of $P(x)$ at a real zero of odd multiplicity crosses the x-axis, while the graph of $P(x)$ at a real zero of even multiplicity touches the x-axis and turns around, that is, it has a turning point at the x-intercept.

7. a) Not a polynomial function because graph is not smooth.
b) Polynomial function

8.



Dividing Polynomials

1. It tells you that the divisor is a factor of the original polynomial, and the quotient is what is left if the divisor is factored out.

2. Yes, otherwise we could continue the long division.

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3. The division could not continue.

4. a) $x^2 + x - 2$ b) $3x + 7 + (26/(x - 3))$
   c) $x^3 + 3x^2 + 9x + 27$ d) $x^2 - 4x + 1 + ((4x - 1)/(2x^3 + 1))$

5. a) $5x^2 + 4x + 11 + 33/(x - 2)$ b) $x + 2$

6. $f(3) = -27$

7. The solution set is $\{-2, \frac{1}{2}, 3\}$

8. 2