PRECALCULUS PROBLEM SESSION #6 SOLUTIONS

Zeros of Polynomial Functions

1. No, the Rational Zero theorem **does not guarantee** that a polynomial function with integral coefficients has any real zeros, it only **gives a list of possible rational zeros**, i.e. possible rational zeros come from the list of the ratio of the integer factors of the constant term of the polynomial dividing by the integer factors of the leading coefficient of the polynomial.

2. a) $\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

   b) $\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$

3. No, because for every zero of a polynomial there is a corresponding linear factor and an nth-degree polynomial cannot be factored into more than n factors

4. Yes, because if $P(x) = 0$ for some x, the so does $-P(x)$.

5. a) $\frac{p}{q}: \pm 1, \pm 2, \pm \frac{1}{2}$ and $2, -\frac{1}{2}, 1$ are rational zeros.

   b) $\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ and $4, -3, 1$ are rational zeros.

6. By the Rational Zeros theorem, the possible rational zeros are $\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15$.
   There are two typos in the problem. The solution set is $\{3, -1, -1 + 2i, -1 - 2i\}$.

7. a) True, a polynomial function of degree n has n zeros, provided multiple zeros are counted more than once and provided complex zeros are counted, which appear in pairs. Thus, $P(x)$ has at most 3 real zeros or 1 real zero

   b) True, a polynomial function of degree n has n zeros, provided multiple zeros are counted more than once and provided complex zeros are counted, which appear in pairs.

   c) False, complex zeros appear in complex conjugate pairs.

8. a) If $P(x)$ had rational zeros they would have to be 1 or -1, but $P(1) = -1$, and $P(-1) = -1$, so $P(x)$ has no rational zeros (although it has at least one real zero).

   b) $-\sqrt[3]{k}$ is a real zero of $P(x)$.

9. Because complex zeros of real polynomials come in conjugate pairs. A cubic function must either have on real root or 3 real roots, therefore the graph of cubic function must touch or cross the x-axis at least once.

10. a) $f(x) = x^3 - 4x^2 + 9x - 36$

    b) $f(x) = 3x^3 + 12x^2 - 93x - 522$

11. Polynomial functions are a subset of the set of rational functions. They are exactly the rational functions of the form $\dfrac{P(x)}{Q(x)}$ where $Q(x)$ is identically 1.

Rational Functions and Their Graphs

1. Polynomial functions are a subset of the set of rational functions. They are exactly the rational functions of the form $\dfrac{P(x)}{Q(x)}$ where $Q(x)$ is identically 1.

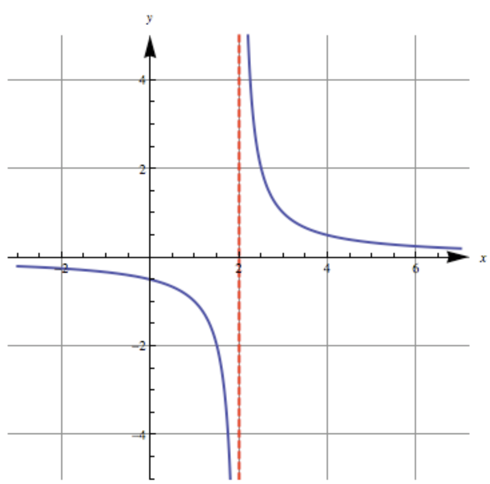
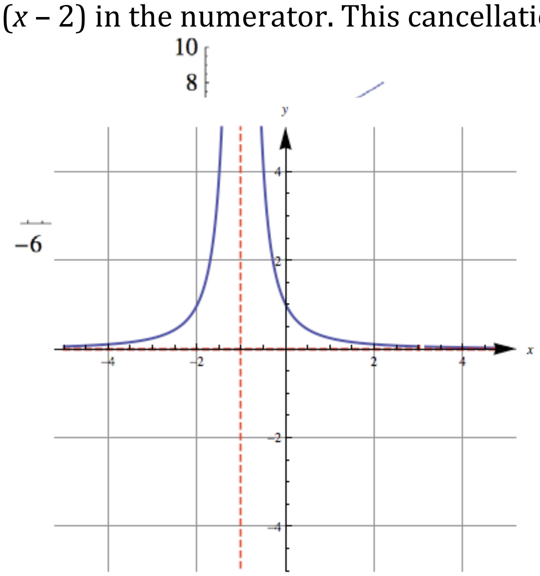
2. A rational function $\dfrac{P(x)}{Q(x)}$, where P and Q are polynomials will be defined over all real numbers if and only if $Q(x)$ has no real zeros.

3. Because both describe the behavior of the function as $x \to \infty$, so either the function approaches zero (horizontal asymptote) or the function approaches ∞ or $-\infty$ (oblique asymptote).

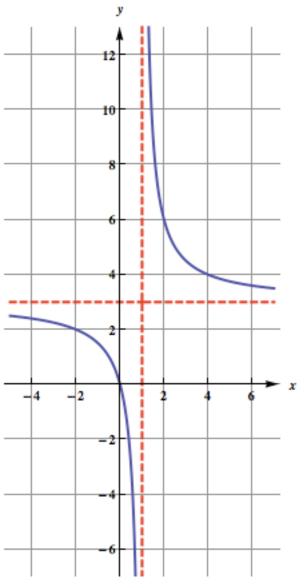
4. $(-\infty, -6) \cup (-6, 2) \cup (2, \infty)$
5. The graph of $g(x)$ has vertical asymptotes at $x = 0$ and $x = 3$.

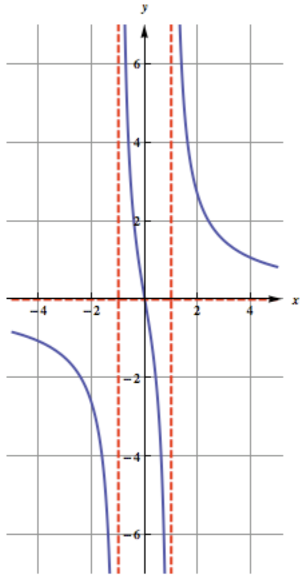
6. The graph of $g(x)$ has a horizontal asymptote at $y = 5$.

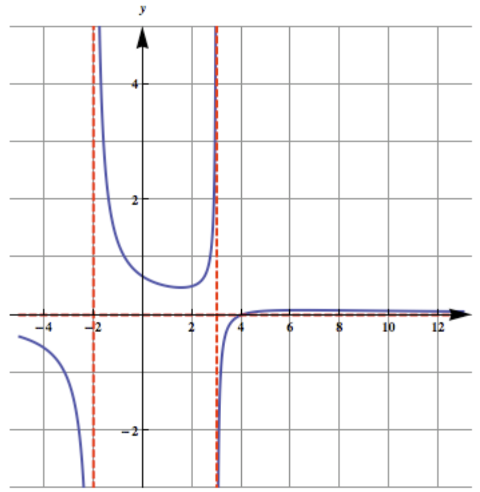
7. No, $x = 2$ is not a vertical asymptote of the graph of $f(x) = \frac{(x^2-4)}{(x-2)}$ because the factor of $(x - 2)$ in the denominator of $f(x)$ cancels with the common factor $(x - 2)$ in the numerator. This cancellation appears in the graph of $f(x)$ as a hole.

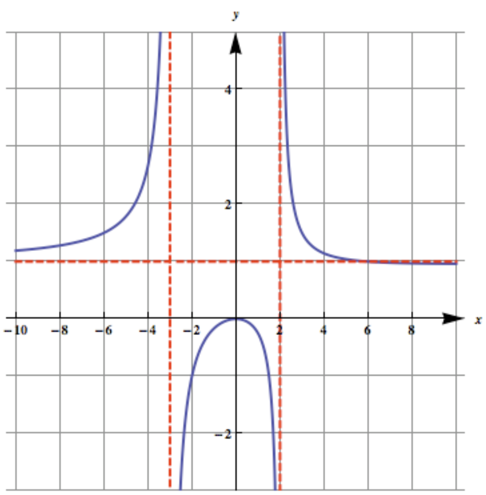
8. a)  b) 

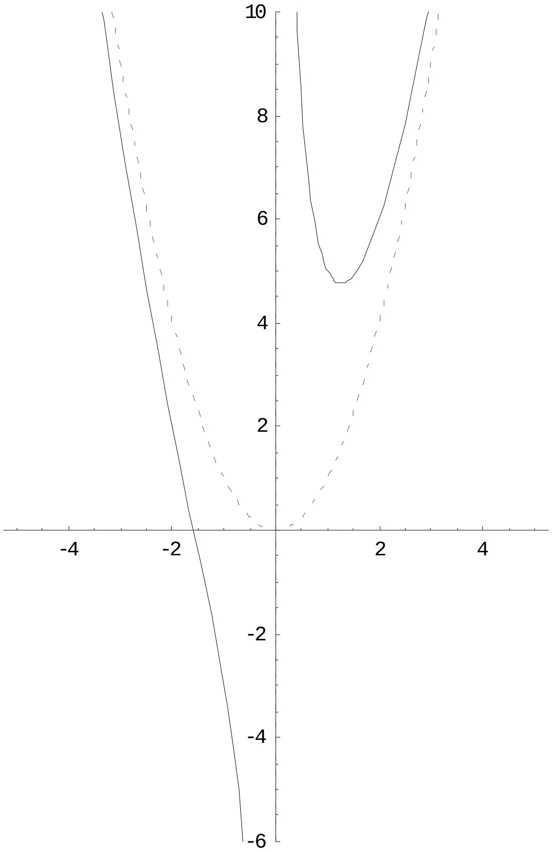
9.

a) 

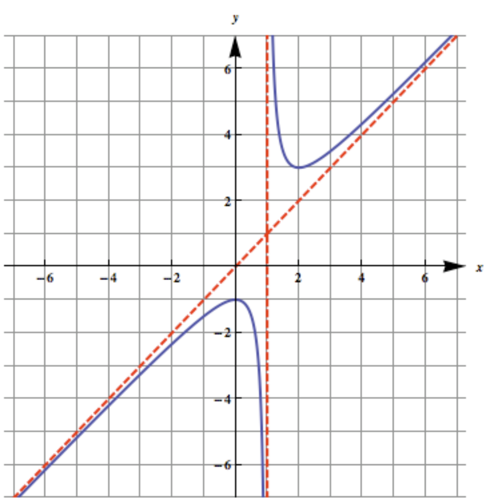
b) 

c) 

d) 

10. 

Notice that as the *x*-values approach -∞ or ∞, the graphs of y_1 and y_2 get closer and closer, that is, in the long run the graph of the rational function approached the graph of the parabola.

11. 

The equation of the slant asymptote of the rational function $f(x)$ is $y = x$.

12. As $x \to -3$, $(f(x)) \to +\infty$.

As $x \to 1$, $(f(x)) \to \infty$.

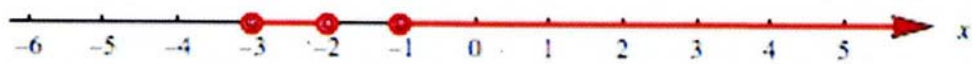
As $x \to \infty$, $(f(x)) \to 0$.

PRECALCULUS PROBLEM SESSION #6 SOLUTIONS

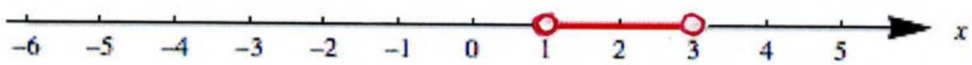
Polynomials and Rational Inequalities

1.

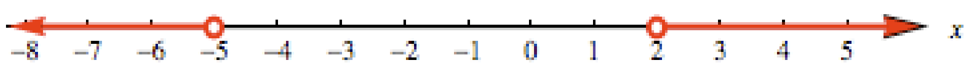
a) $[-3, -2] \cup [-1, \infty)$



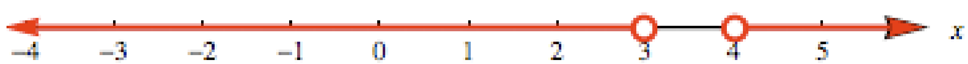
b) $[1, 3]$



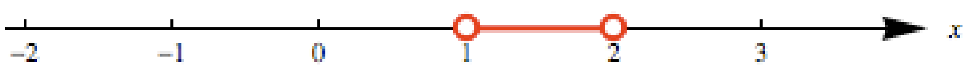
2. a) $(-\infty, -5) \cup (2, \infty)$



b) $(-\infty, 3) \cup (4, \infty)$



c) $(1, 2)$



3. Because a sign change can occur at the x-intercepts or at the location of the vertical asymptotes of a rational function.

4. Domain of $f(x) = D_{f(x)} = (1/2, 1]$

5. $x^2 + x - 12 < 0$ OR $(x + 4)(x - 3) < 0$

6. $x^3 - 6x^2 - 19x + 84 > 0$ OR $(x + 4)(x - 3)(x - 7) > 0$

7. $x < 2$ or $(-\infty, 2)$