Exponential Functions

1. The function $y = 2x$ is a linear function and the function $y = 2^x$ is an exponential function, with the following graphs. Notice, however, that both share the point (1, 2).
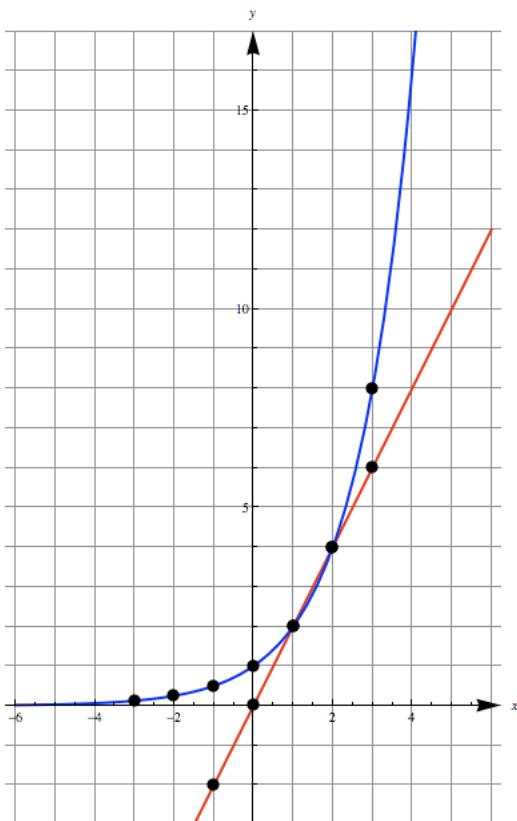
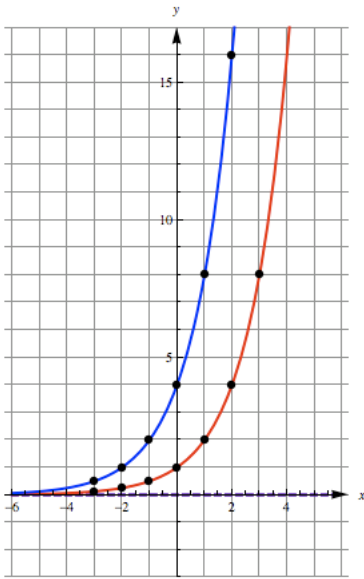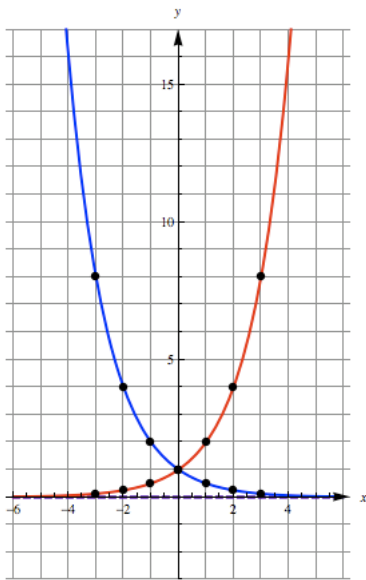2. In the following graphs the red graph is the graph of the original function, $y = 2^x$.
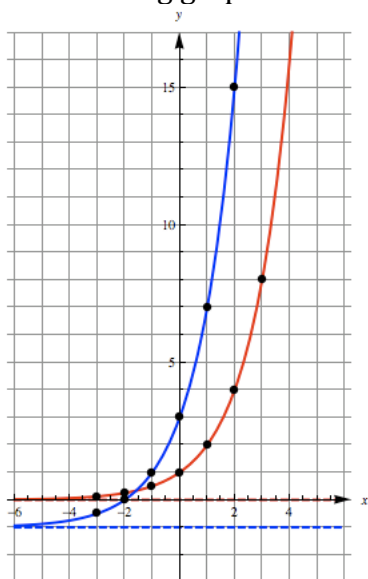
$h(x)$ has a horizontal asymptote at $y = -1$.
The domain of $h(x)$ is $D_{h(x)} = (-\infty, \infty)$ and the range of $h(x)$ is $R_{h(x)} = (-1, \infty)$.

a)

$g(x)$ has a horizontal asymptote at $y = 0$.
The domain of $h(x)$ is $D_{g(x)} = (-\infty, \ \infty)$ and
the range of $h(x)$ is $R_{g(x)} = (0, \ \infty)$.

b)

$g(x)$ has a horizontal asymptote at $y = 0$.
The domain of $g(x)$ is $D_{g(x)} = (-\infty, \ \infty)$ and
the range of $g(x)$ is $R_{g(x)} = (0, \ \infty)$.

c)

3. a) $g(x) = 3^{x-1}$ b) $f(x) = 3^x$ c) $g(x) = 3^{-x}$

4. $f(x) = 1000\,(0.5)^{\frac{80}{30}} = 157.49$ Chernobyl will not be safe for human habitation by 2066.
There will still be 157.5 kilograms of cesium-137 in Chernobyl's atmosphere.

5. a) $24\left(1 + \dfrac{0.05}{12}\right)^{12\,(379)} \approx \$3,\,917,\,360,\,753$

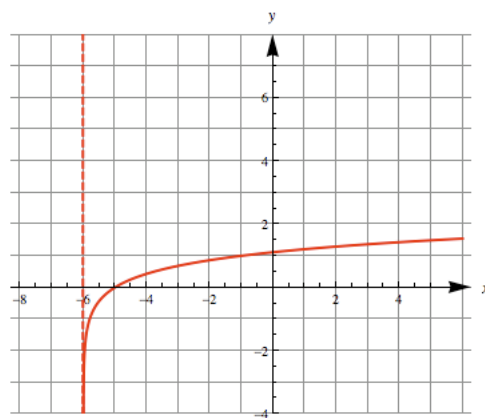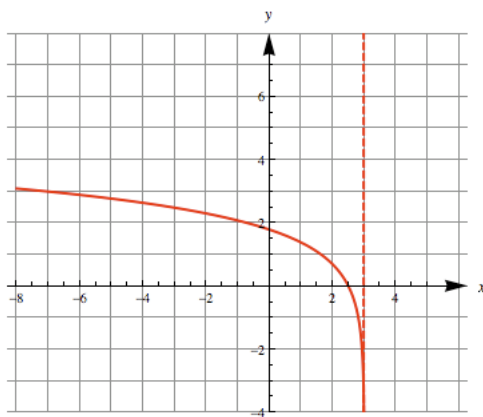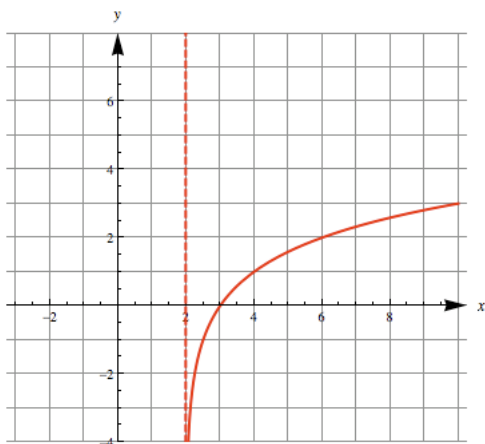b) $24\,e^{0.05\,(379)} \approx \$4,\,074,\,662,\,794$

6. $7.6 \approx 8$ years

7.   If $b = 1$, then for any x, $f(x) = b^x = 1^x = 1$, thus making the exponential function into a constant function.
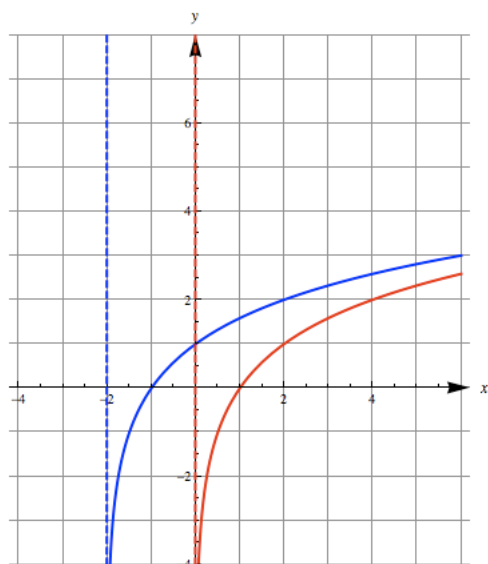
Logarithmic Functions

1. $\log_{64} 4 = \frac{1}{3}$

2.   (a) –2   (b) 2   (c) –1   (d) 1/2   (e) 1   (f) 0   (g) 5   (h) 7

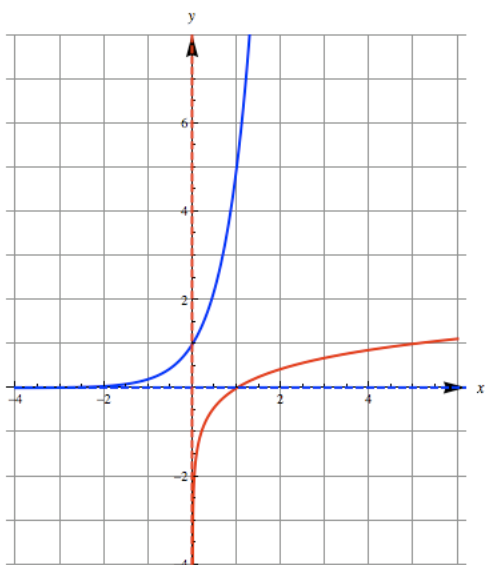3.   (a) $g(x) = \log_2(x - 2)$, domain: $x > 2$ (b) $h(x) = \ln(6 - 2x)$, domain: $x < 3$

(c) $f(x) = \log_5(x + 6)$, domain: $x > -6$

4. The graph of $f(x)$ is in red and the graph $g(x)$ is in blue.

5. $0 < b < 1$

6. The graph of $f(x)$ is in blue and the graph $g(x)$ is in red.

7. a) $f(x)$

    b) $G(x)$

    c) $F(x)$

8. 124.7% if $f(x) = 62 + 35\log(x - 4)$.
    But, if $f(x) = 6.2 + 35\log(x - 4)$ then 68.9%.