Normal Distribution/Empirical Rule/Z-Scores Word Problems

1. The average playing time of CDs in a large collection is 35 minutes, and the standard deviation is 5 minutes.
   a. What value is 1 standard deviation above the mean? 1 standard deviation below the mean? What values are 2 standard deviations from the mean?
   b. Assuming the distribution of time is approximately normal, about what percentage of times are between 25 and 45 minutes? Less than 20 minutes or greater than 50 minutes? Less than 20 minutes?

   a) 40 minutes, 30 minutes, 25-45 minutes
   b) 95%, 0.3%, 0.15%

2. Suppose your statistics professor returned your first midterm exam with only a z-score written on it. She also told you the histogram of the scores was approximately normal. How would you interpret each of the following z-scores?
   a. 2.2
   b. -0.4
   c. 0

   a) You did very well!
   b) You did slightly below average.
   c) You did average.

3. The test scores for the quantitative reasoning section of the GRE are normally distributed. The mean score is 150 with a standard deviation of 8.75. The test scores of four students are 162, 168, 155, and 138. Find the z-score that corresponds to each value.

   Score $162 \rightarrow z = 1.37$ Score $155 \rightarrow z = 0.57$

   Score $168 \rightarrow z = 2.06$ Score $138 \rightarrow z = -1.37$

4. In a recent study on world happiness, participants were asked to evaluate their current lives on a scale from 0 to 10, where 0 represents the worst possible life and 10 represents the best possible life. The responses were normally distributed, with a mean of 5.4 and a standard deviation of 2.2. Find the probability that a randomly selected study participant's response was:
   a. Less than 4
   b. Between 4 and 6
   c. More than 8

   a) 0.2611
   b) 0.3453
   c) 0.1190

5. The red blood cell count for a population of adult males can be approximated by a normal distribution, with a mean of 5.4 million cells per microliter and a standard deviation of 0.4 million cells per microliter. What is the minimum red blood cell count that can be in the top 25% of counts? What is the maximum red blood cell count that can be in the bottom 15% of counts?

    5.672 million, 4.984 million

6. A vending machine dispenses coffee into an 8-ounce cup. The amounts of coffee dispensed into the cup are normally distributed, with a standard deviation of 0.03 ounces. You can allow the cup to overflow 1% of the time. What amount should you set as the mean amount of coffee to be dispensed?

    $\mu = 7.93 \ ounces$

7. A large population has a mean of 310 and a standard deviation of 37. Draw out a normal curve and label the values at 1, 2, and 3 standard deviations away from the mean. Then, using the empirical rule, find what percentage of the observations fall in the intervals $\mu \pm 1\sigma$, $\mu \pm 2\sigma$, and $\mu \pm 3\sigma$.
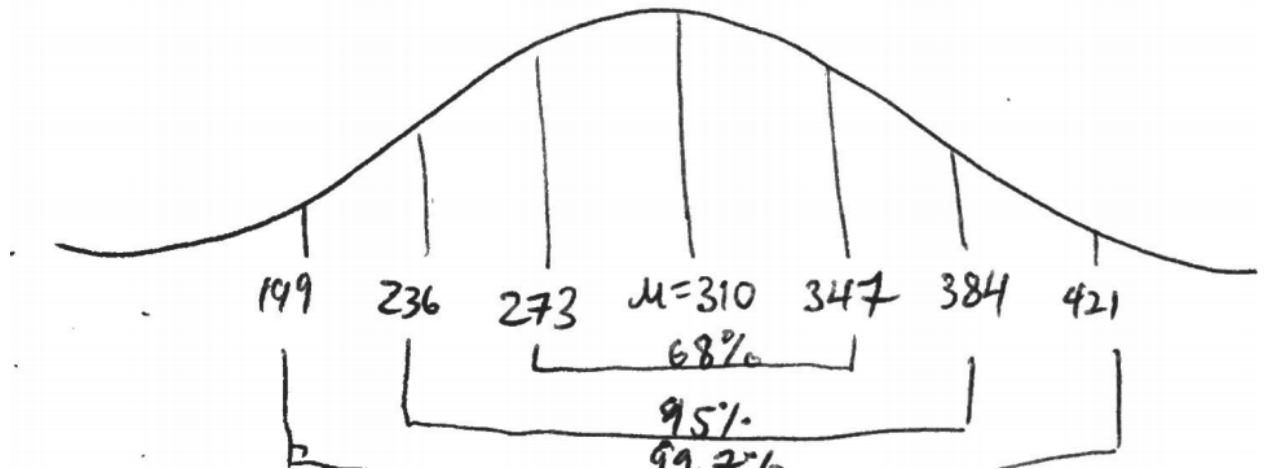
8. The length of human pregnancies from contraception to birth varies according to a distribution that is approximately normal with mean 266 days and standard deviation 16 days. Use the empirical rule.
    a. Between what values do the lengths of the middle 95% of all pregnancies fall?
    b. How short are the shortest 2.5% of all pregnancies? How long do the longest 2.5% last?
        a) 234 days and 298 days
        b) Shorter than 234 days, longer than 298 days

9. The heights of women aged 20 to 29 are approximately normal with mean 64 inches and standard deviation 2.7 inches. Men the same age have mean height 69.3 inches with standard deviation 2.8 inches. What are the z-scores for a woman 6 feet tall and a man 6 feet tall? What information do the z-scores give that the actual heights do not?

    6 ft. woman $\rightarrow z = 2.96$
    6 ft. man $\rightarrow z = 0.96$
    Being 6 ft. tall is more extreme for a woman than a man.

10. The scale of scores for an IQ test are approximately normal with mean 100 and standard deviation 15. The organization MENSA, which calls itself the "high IQ society", requires a score of 130 or higher. What percent of adults would qualify for membership?

    2.5%