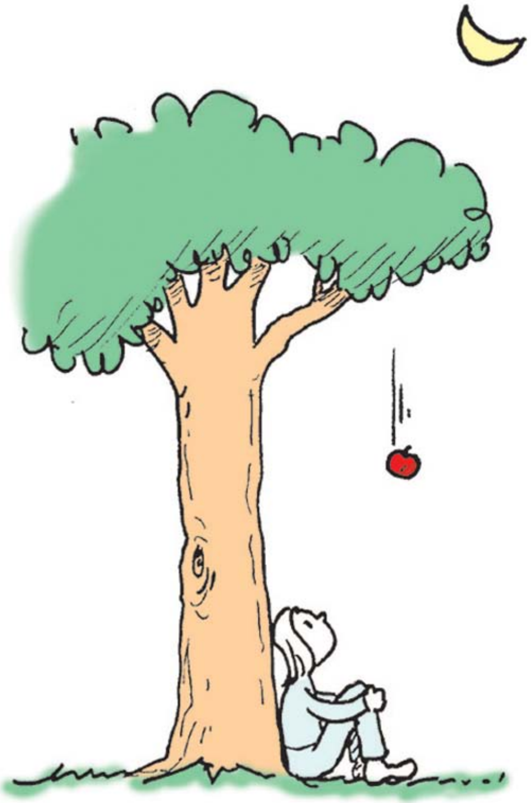
Today:

Chapter 9 (Gravity)

Chapter 9: Gravity

Newton: made revolutionary connection between the circular motion of celestial bodies and the downward falling of objects on the earth:

It is the one and the same gravitational force responsible for both the apple falling from the tree and the moon orbiting around the earth!



The universal law of gravity (Newton)

- Every mass m_1 attracts every other mass m_2 with a force:

$$F \sim \frac{m_1 m_2}{d^2}$$

distance between their centers

The greater (either of) the masses, the greater is the attractive force.

The closer they are to each other, the greater the force – with an inverse-square dependence.

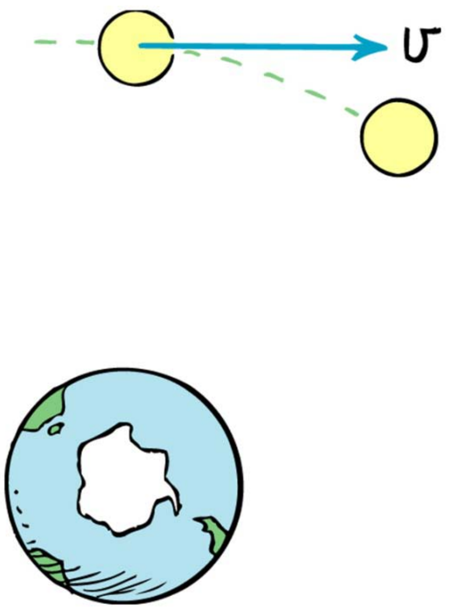
- The constant of proportionality is called the **universal gravitational constant,** G = 6.67 x 10 $^{-11}$ N . m^2/kg^2 = 0.0000000000667 N m^2/kg^2

Tiny! So gravitational forces between everyday masses at everyday distances (eg you and me) is negligible.

$$F = \frac{G\, m_1 m_2}{d^2}$$

Clicker Question

If there was no earth (and no other planets/sun…), the moon would continue going in a straight line as shown by the solid arrow. The gravitational pull of the Earth draws the moon closer to it, hence it falls in an orbit around the earth, rather than directly into it.



What would happen if the tangential speed of the moon was instead zero?

A) It would still continue orbitting the Earth

B) It would be stationary with respect to the Earth.

C) It would fall straight down into the Earth….crash!

D) None of the above

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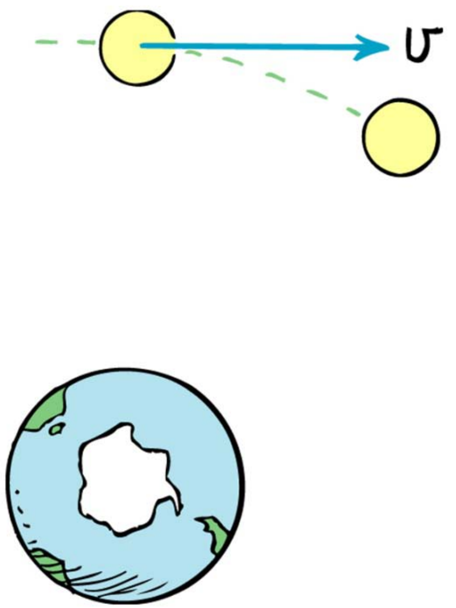

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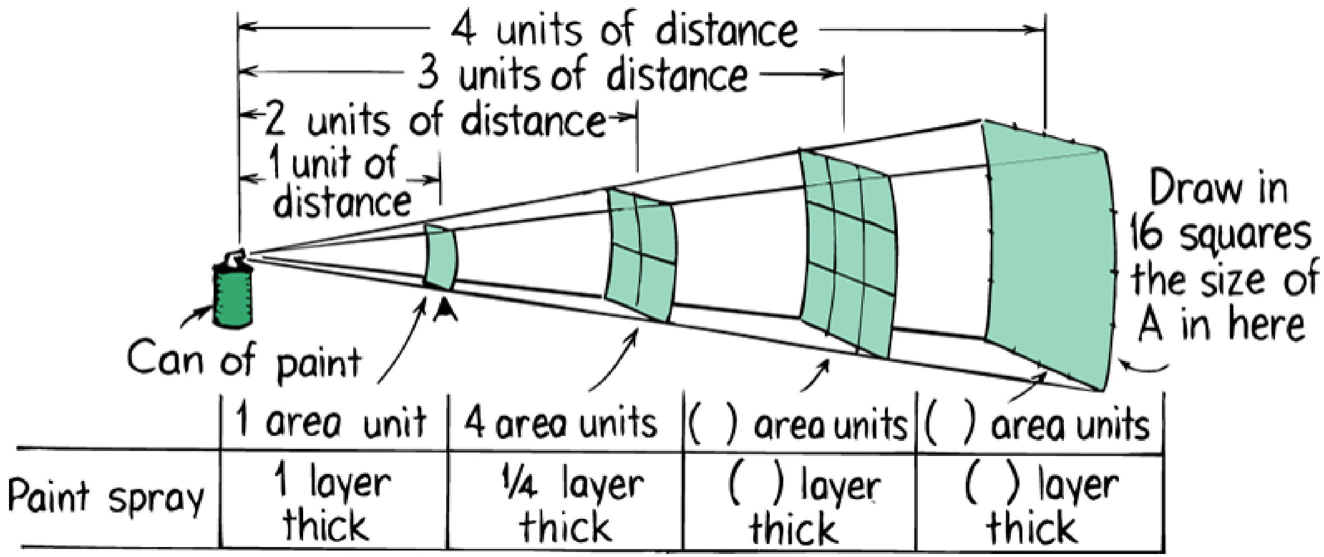
Answer: C, due to the gravitational force of the Earth on the moon

Distance-dependence of gravity

- Inverse-square law: $F \sim 1/d^2$

Compare with paint-spray burst out from a can: the thickness of the paint varies in the same inverse-square way i.e. if 1-layer thick at 1m, then is ¼ layers thick at 2 m etc.



Distance-dependence continued…

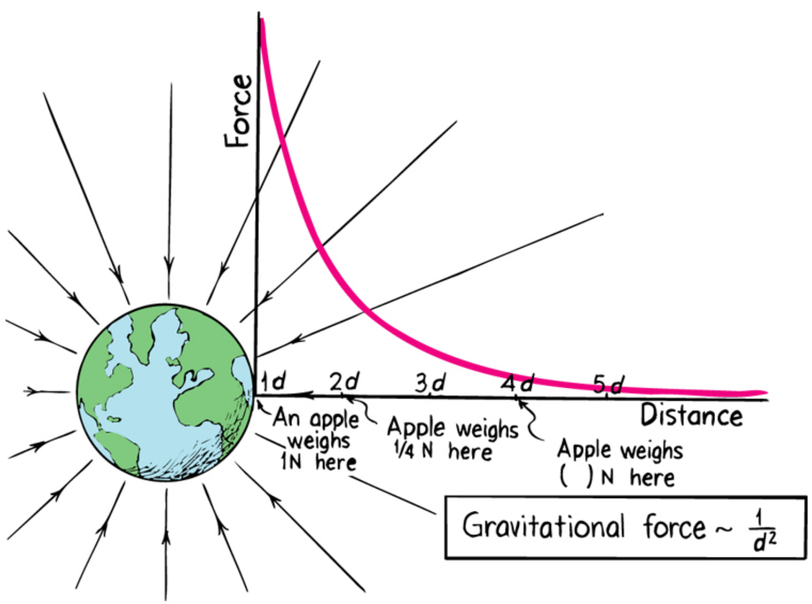
Notes (1) d = distance between the *center of masses* of the objects.

So when one of the objects is earth, then the relevant distance

d = radius of the earth + distance of other object from earth's surface.

6.4×10^6 m

(2) Even very very far from earth, its gravitational force is never actually zero, but it does decrease rapidly and forces from other more nearby objects would overwhelm the grav force from earth.



Questions

(1) What is the force of earth's gravity on a 1-kg object at the surface of the earth? What do we commonly call this force?

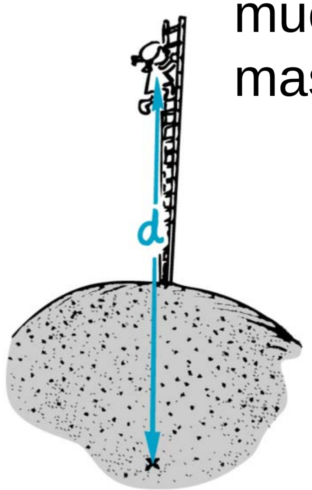
$F = G\, m_{earth}\, m_{1kg}/d_{earth}^{\,2}$

$= (6.67 \times 10^{-11})(6 \times 10^{24}\ kg)\ (1kg)/(6.4 \times 10^{6}\ m)^2 = 9.8\ N$

The force of gravity on an object is how we defined its weight.

i.e. $g = 9.8$ N/kg that we defined earlier, is just $g = Gm_{earth}/R_{earth}^{\,2}$. Ordinary distances on earth are so small c.f. radius of earth, that their distance to earth's center is ~ R_{earth}, so grav force on them is just mg.

(2) If you climbed to the top of Mount Everest (height 8850 m), how much less would you weigh? Assume you eat on the way so that your mass remains fixed.



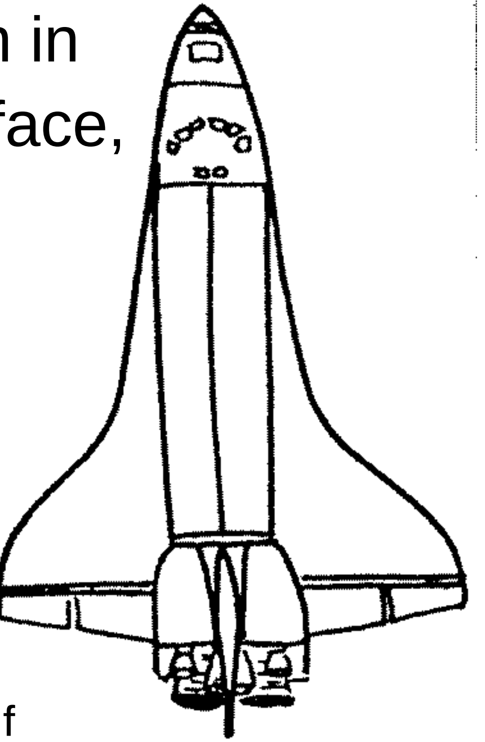
At top of Everest, $d = 6.4 \times 10^{6} + 8850 = 6.40885 \times 10^{6}$ m

So, the force is $(6.4/6.40885)^2 = 0.997$ as much

eg. If you weigh 200-lb here, then you'll weigh 199.4-lb on Mt Everest.

Clicker Question

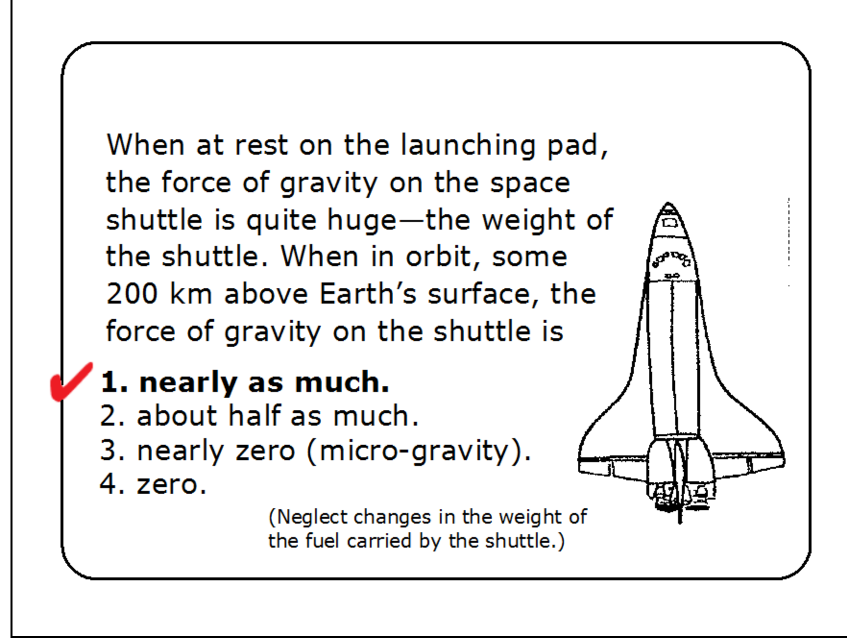
When at rest on the launching pad, the force of gravity on the space shuttle is quite huge—the weight of the shuttle. When in orbit, some 200 km above Earth's surface, the force of gravity on the shuttle is



1. nearly as much.
2. about half as much.
3. nearly zero (micro-gravity).
4. zero.

(Neglect changes in the weight of the fuel carried by the shuttle.)

Answer:1, nearly as much



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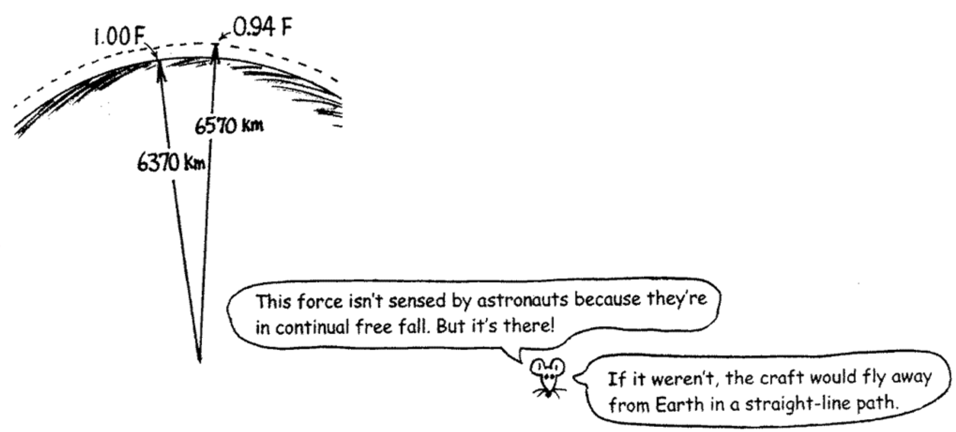
(Neglect changes in the weight of the fuel carried by the shuttle.)

The gravitational force on the shuttle, whether at rest or in orbit, depends on only three things: its mass, the mass of Earth, and its distance from Earth's center. The only variable is distance. On the launching pad the shuttle is about 6370 km from Earth's center. When in orbit it is about 6370 + 200 km from the Earth's center. In accord with $F = GmM / R^2$ the 200-km difference in distance means a 0.06 fractional difference in force. Discounting the changes in the fuel, the gravitational force on the shuttle in orbit is 94% as much as when on Earth's surface—nearly the same.



Question:

Jupiter is about 300 times as massive as the earth but with radius about 11 as much as that of earth. On which would an apple weigh more ?

$$F = \frac{G\, m_p\, m_a}{d^2}$$

where m_p is mass of the planet and m_a is mass of the apple

So on Jupiter $F_{\text{on apple}} = G\, m_a(300 m_E)/(11 R_E)^2$

where m_E and R_E are the mass and radius of Earth

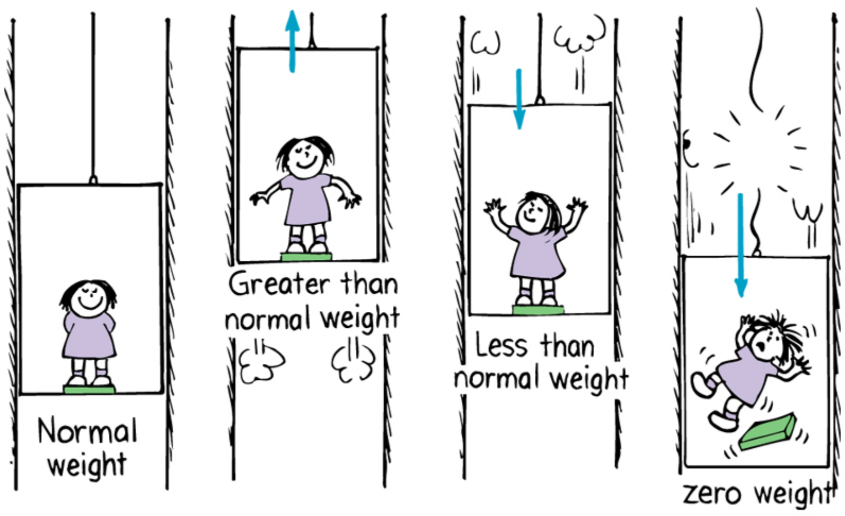
$= (300/11^2) G\, m_a m_E / R_E^2$

$= 2.6\, F_{\text{on apple}}$ on Earth

→ Apple weighs 2.6 times more on Jupiter than on Earth

Weight and Weightlessness

- Earlier, we defined weight as force due to gravity, *mg*.
- But if we *accelerate*, we may “feel” heavier or lighter – eg. in an elevator:



Your “apparent weight” depends on your acceleration

If the elevator accelerates upwards, any scales you are standing on will read a higher weight and you feel heavier; if accelerates downwards, they read a lower weight and you feel lighter.

The scales measure how much a spring inside is compressed – i.e. how much force it must exert to balance (or support) the force you are exerting on it.

Weight/weightless continued…

- We will now **define apparent weight** to measure this instead --

Define apparent weight = force exerted against a supporting surface or a weighing scale.

(Note: your textbook calls "apparent weight" just weight at this point!)

Then, you are as heavy as you feel ! (c.f. elevator again)

If the elevator is in free fall (cable broken), then your apparent weight is zero, since there is no support force. "Weightless".

Gravity is still acting on you, causing downward acc. but not felt as weight.

Same weightlessness for astronaut in orbit – he still has gravity acting on him, but since every object in his shuttle (including any bathroom scale) is falling around the earth with him, he is not supported by anything, no compression in the scales etc.

Clicker Question

Inside a free-falling elevator, there would be no

A) gravitational force on you
B) apparent force on you
C) both of these
D) none of these

Clicker Question

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Answer: B
The gravitational force on you is what we call your weight, mg, provided by your gravitational interaction with the earth. However you feel weightless because there is no support force when you are in free-fall – there is therefore no apparent force.

<u>Ocean Tides</u>

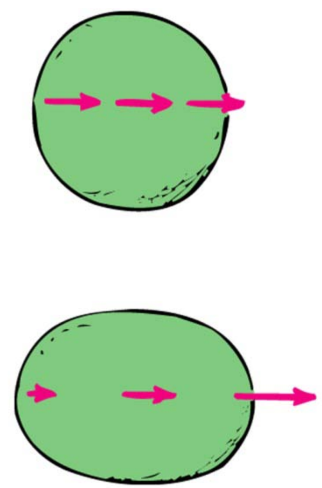
- Caused by <u>differences</u> in the gravitational pull of the moon on the earth on opposite sides of the earth.
- Moon's pull is stronger on the side of the earth that it is closest to; weakest on the opposite side, because F decreases with distance.

- Why does this result in *two* high-tides (and two low-tides) every day? Because when the moon is **either** closest or farthest away, you get a maximum bulge:

Imagine earth to be a ball of jello.

If moon's force was equal at every point, then it all accelerates together towards moon.

But moon's force is actually more like arrows here: so ball gets elongated – *both sides* effectively bulge.



(moon over here somewhere)

Tides continued

So, *relative to the moon*, the tidal bulges remain fixed while Earth spins beneath – mostly it is the oceans that bulge out equally on opposite sides, on average nearly 1-m above.

Note: the moon’s pull on the earth is equal and opposite to the earth’s gravitational pull on the moon. Centripetal force.

More on tides…

- Since earth spins once a day, any point on earth has two high tides and two low tides (on average, 1-m below average) a day.

If moon was not orbiting, then the high-low tide separation would be ¼ day, ie. 6 hours.

- But since while the earth spins, the moon moves in its orbit, it turns out the moon returns to same point in the sky every 24 hours and 50 minutes – ¼ of this is what determines the high-low-tide time difference.

- This is why high tide is not at the same time every day
- Why are there no tides in lakes?
  - Because lakes are localized; no part of the lake is a lot closer to the moon than any other part, so no big differences in moon's pull in a lake, as opposed to the oceans which span the globe…

Clicker Question

Ocean tides are produced by the Moon. Since our bodies are mostly water, doesn't the Moon similarly produce tides in our bodies?

1. Yes, there are biological tides that affect mood and behavior.
2. Yes, but negligible (less than are produced by an apple you hold over your head).
3. No, because the water in our body isn't free to flow.

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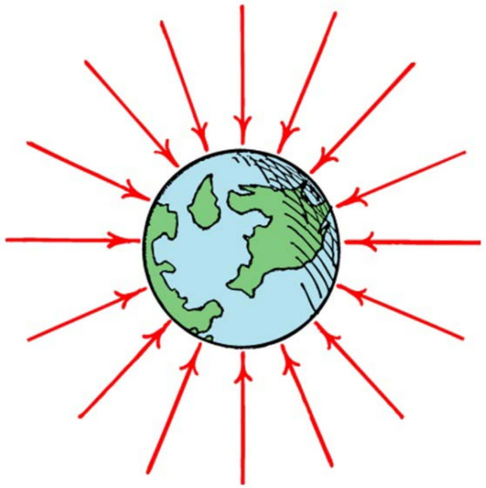
3. No, because the water in our body isn't free to flow.

Answer: 2, yes, but negligible

Tides are caused by *differences* in gravitational pulls by the Moon (or other celestial bodies) that stretch Earth's oceans. The key to tides is differences in pulls, which is related to differences in distance between various parts of a body and the Moon. Earth's ocean tides are the result of thousands of kilometers difference in distance between near and far parts of the ocean. Scarcely any tides occur in a lake because no part is significantly closer to the Moon than other parts. Likewise for the fluids in your body. You're not tall enough for your head to be appreciably closer to the Moon than your feet. The Moon does produce microtides in your body, however. How strong? Less than an apple held a half meter over your head produces!

Gravitational Fields

- Gravitational force acts at a distance – i.e. the objects do not need to touch each other.
- We can regard them as interacting with the *gravitational field* of the other: think of this existing in the space around an object, so another object in this space feels a force towards it.



Field lines have arrows indicating direction of force at that point, and are closer together when the field is strongest.

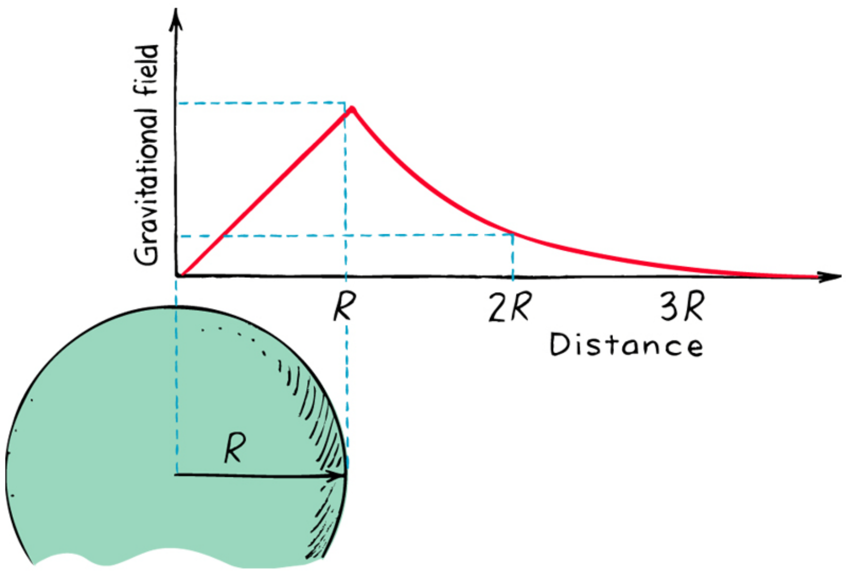
The gravitational field is a vector, same direction as the force, and strength is the force on a mass m, divided by that m:

$g = F/m$, units are N/kg

(Gravitational field inside a planet)

- We will not cover this much or examine this in this course.

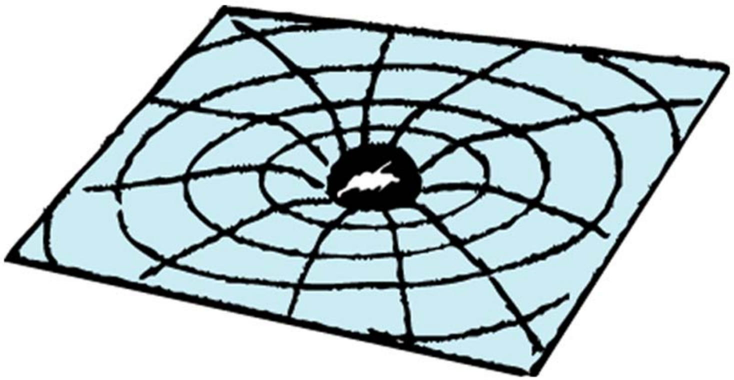
- The only thing we will note is that the field increases linearly inside the planet (and falls off in the usual inverse-square way outside). It is zero right in the middle of the planet.



(A very little on Einstein's Theory of Gravitation)

- 1900's: Einstein's theory of general relativity involves **curved four-dimensional space-time**

Replace bodies producing gravitational fields with warped space-time.



A little on Black Holes

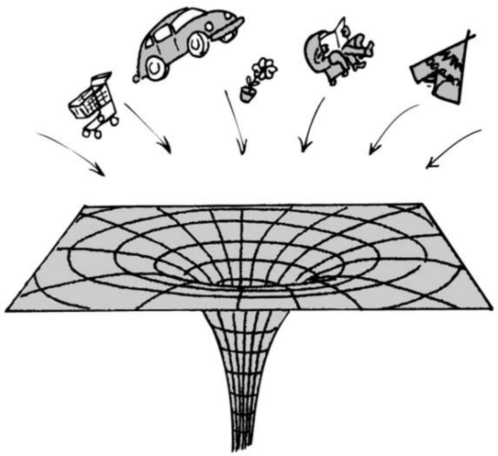
- Because grav force increases with decreasing distance, then if a massive object somehow shrinks tremendously (keeping amount of mass fixed) the grav force <u>on its surface</u> gets tremendously stronger.

- Happens for massive stars (> 1.5 of mass of our sun) when they have burnt their fuel – the stuff left condenses into an extremely dense object (neutron star) which, if large enough, continues to shrink because of its gravity.

- Consider an object on the surface of such a star – it feels increasing grav force, to the point that it can never leave it.

i.e. the speed required to overcome the grav force becomes faster than the speed of light, and *no* object can have such a speed. Called a **black hole**.

This means no object, not even light, can escape from a black hole. Anything coming near gets sucked in and destroyed (although its mass, ang mom, charge are preserved)



Black holes continued…

- Since black holes are invisible, how do we know they exist?

By their grav. influence on neighboring stars – eg binary star systems, where have one luminous star and a black-hole orbiting each other.

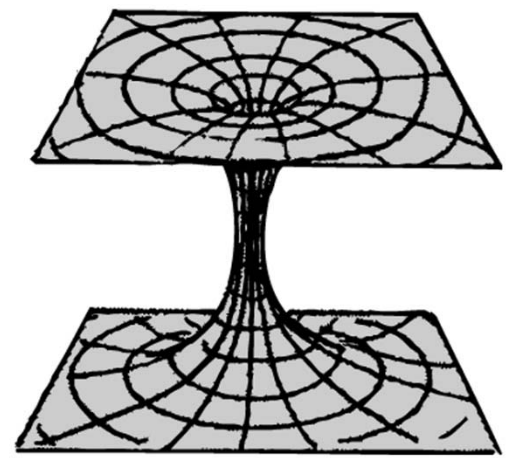
Other experimental evidence indicates massive black holes at the center of many galaxies eg in old ones, stars circle in a huge grav field, with an "empty-looking" center.

Galactic black holes have masses $10^6 - 10^9$ times that of our sun.

- Related, but still speculative, entity: **wormhole**

Instead of collapsing to a point, it opens out again in another part of the universe – time travel…

But still speculative (unlike black holes)



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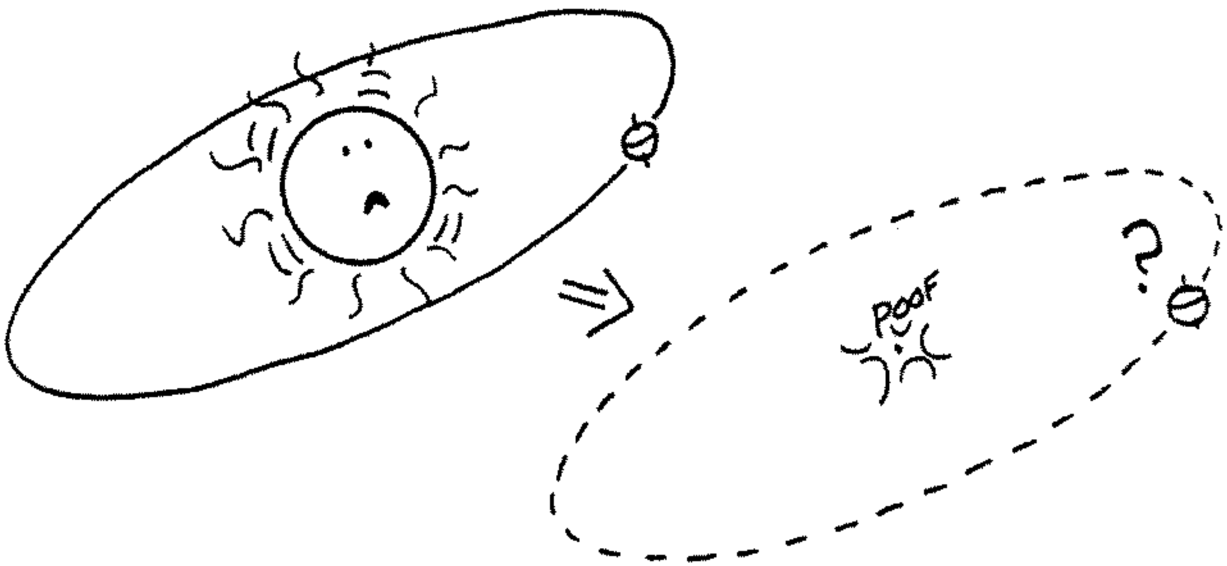
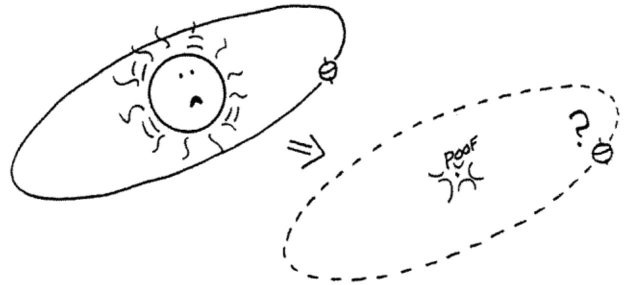


If the Sun suddenly collapsed to become a black hole, the Earth would

1. leave the Solar System in a straight-line path.
2. spiral into the black hole.
3. continue to circle in its usual orbit.

Answer: 3; continue to circle in usual orbit



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2. spiral into the black hole.
3. **continue to circle in its usual orbit.**

We can see from Newton's equation, $F = G\frac{mM}{d^2}$ that the interaction F between the mass of the Earth and the Sun doesn't change. This is because the mass of the Earth does not change, the mass of the Sun does not change even though it is compressed, and the distance from the centers of the Earth and the Sun, collapsed or not, does not change. Although the Earth would very soon freeze and undergo enormous surface changes, its yearly path would continue as if the Sun were its normal size.