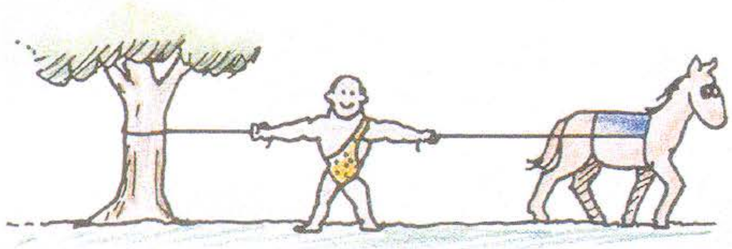
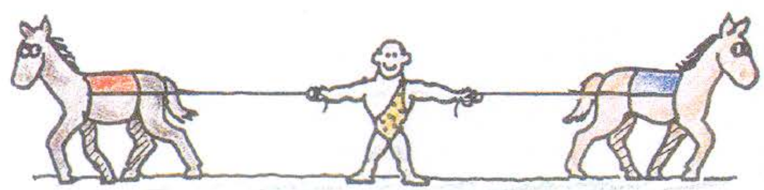
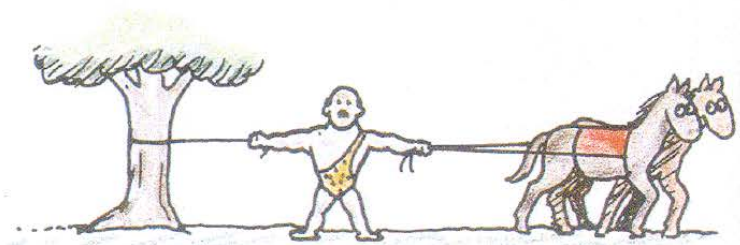
NEXT-TIME QUESTION



The strong man can withstand the tension forces exerted by the pair of ropes—one tied to a tree and the other to a horse. No problem. Compare the tension he experiences in two other situations shown to the right—horse and horse, and a tree and two horses.

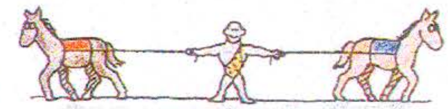




Hewitt Drew it!

NEXT-TIME QUESTION

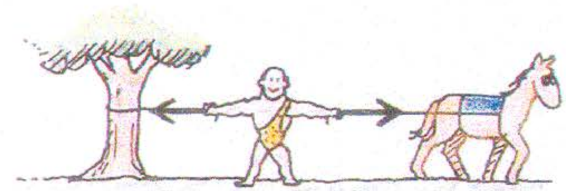
The strong man can withstand the tension forces exerted by the pair of ropes—one tied to a tree and the other to a horse. No problem. Compare the tension he experiences in two other situations shown to the right—horse and horse, and a tree and two horses.

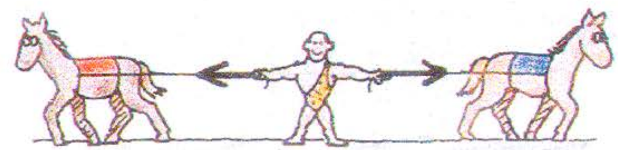


Answer:

The tension is the same whether two horses pull in opposite directions, or one horse pulls to the right and the tree pulls to the left. Does the tree really pull? If it didn't, wouldn't the strong man be moved by the horse?

When two horses pull in the same direction, however, tension doubles. In this case, the tree also pulls twice as hard. If it didn't, would the strong man remain stationary?





If the tree didn't pull on the rope, the rope couldn't pull on the tree. That's Newton's third law!

Hewitt Drew it!

Chapter 6: Momentum

Momentum = “inertia in motion”

Specifically, momentum = mass x velocity

$$\boxed{= m\,v}$$

Eg. Just as a truck and a roller skate have different inertia, when they are moving, they (generally) have different momenta.

Question: (i) Does the truck always have more inertia than the roller skate?

(ii) What about momentum?

(i) Yes (mass larger)

(ii) No – eg a roller skate rolling has more momentum than stationary truck. Momentum depends on speed as well as mass .

Impulse

- How can the momentum of an object be changed?

By changing its mass, or, more usually, its velocity – i.e. by causing an acceleration.

What causes acceleration?

A force

But the time over which the force acts, is also important. Eg. If trying to get a broken down car moving, and you push tremendously but only for a split-second, it won't move. You need to exert the force for a longer time.

- The "effectiveness" of the force in causing a change in momentum is called the impulse:

$$\boxed{\begin{aligned}\text{Impulse} &= \text{force x time interval}\\ &= F\,t\end{aligned}}$$

How exactly is the momentum changed? Use N's 2nd law, $a = F/m$, or, $F = ma$.

So, impulse = $ma\,t$

$= m$ *(change in velocity/time) time* $= m$ *(change in velocity)*

i.e. $\boxed{\begin{aligned}&\text{impulse = change in momentum}\\ &Ft = \Delta\,(mv)\end{aligned}}$

Using the impulse-momentum relation

Increasing momentum

- As highlighted by the broken-down car example, need to apply large force for a large time.

- Eg. The longer the barrel of a cannon, the greater the speed of the emerging cannonball because the forces on it from the expanding gasses have more time to act.

- Eg. Why does an archer pull his arrow all the way back before releasing it?

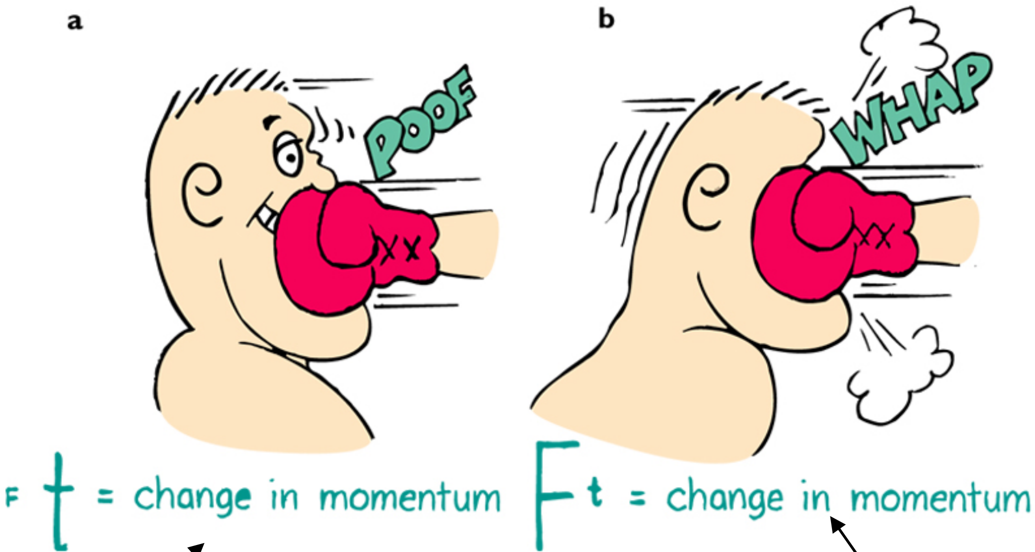
To give more time for the (time-varying) elastic force of the bow to act, so imparting greater momentum.

Decreasing momentum over a long time

Often you want to reduce the momentum of an object to zero but with minimal impact force (or injury).

→ try to maximize the time of the interaction (remember *Ft* = Δ (*momentum*))

Eg. Riding with the punch, when boxing, rather than moving into it…



Here, by moving away, the time of contact is extended, so force is less than if he hadn't moved.

Here, by moving into the glove, he is lessening the time of contact, leading to a greater force, a bigger ouch!

Decreasing momentum over a long time – more examples:

Eg. Car crash on a highway, where there's either a concrete wall or a barbed-wire fence to crash into. Which to choose?
Naturally, the wire fence – your momentum will be decreased by the *same* amount, so the impulse to stop you is the same, but with the wire fence, you extend the time of impact, so decrease the force.

Eg. Bend your knees when you jump down from high! Try keeping your knees stiff while landing – it hurts! (only try for a small jump, otherwise you could get injured…) Bending the knees extends the time for momentum to go to zero, by about 10-20 times, so forces are 10-20 times less.

Eg. Safety net used by acrobats, increases impact time, decreases the forces.
Try dropping an egg into a suspended cloth, rather than hitting the floor.

Eg. Catching a ball – tend to let your hand move backward with the ball after contact…

Question

a) Is the impulse to stop a 10 kg bowling ball moving at 6 m/s less, greater or the same, if it is done in 1s rather than 2s?

Same, since impulse = change in momentum is the same whatever the time it takes.

b) Is the force you must exert to stop it less, greater, or the same, if done in 1s or 2s?

Twice as great force if you do it in 1s than if you do it in 2s, because change in momentum = impulse = $F\Delta t$. (so half Δt means twice F)

c) In a general situation, when does impulse equal momentum?

If the object's initial momentum is zero, then

impulse = momentum change = final mom. – ~~initial mom.~~ = final momentum.

Likewise, if object is brought to rest, then impulse = - initial momentum.

Decreasing momentum over a short time

On the other hand, sometimes the object is to obtain large forces when decreasing momentum. Want short impact times.

Eg. This is how in karate (tae kwon do), an expert can break a stack of bricks with a blow of a hand: Bring in arm with tremendous speed, so large momentum, that is quickly reduced on impact with the bricks. The shorter the time, the larger the force on the bricks.

Bouncing

Why is it that if the expert makes her hand bounce back upon impact, she can increase the force on the bricks?

Because, bouncing means reversing of momentum, so even greater momentum change.

Eg, Say 1-kg object at 1-m/s comes to rest. Then Δ(mom) = -1 kg m/s

Say instead it bounces back at 1 m/s. The change in momentum is then

-1-(1) =-2kg m/s

(Don’t be fazed by the – signs, they just indicate direction – the point is that the size of the change is larger in the bouncing case)

Momentum conservation

- First distinguish: **internal forces** vs **external forces** on system

Are interactions within the system

Eg. For baseball, molecular forces holding ball together

Eg. Rifle+bullet system, then the forces between rifle and bullet are internal

Are interactions with objects not part of system

Eg. Bat's hit on the ball is external to the ball

Eg. For rifle+bullet as system, external forces are gravity, and support force of what it is resting on. (If rifle is not moving vertically, these cancel…)

So, what is internal and what is external depends on what we choose to include in the system.

• To change the system's momentum, need a net external force. (from 2nd law)

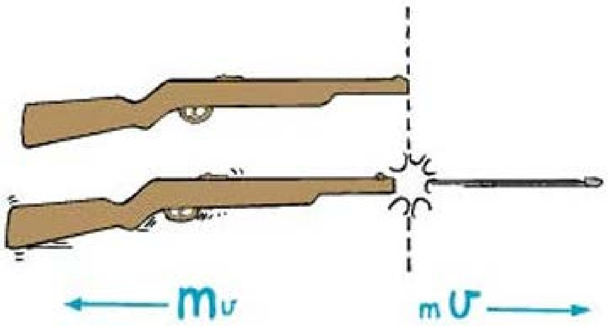
• Equivalently, if no net external force, can be no momentum change.

i.e. **momentum is conserved if $\boldsymbol{F_{net,ext} = 0}$.**

Eg. Rifle(R) + bullet(b)

- When bullet (b) is fired from rifle(R), there are no net external forces, so momentum of the rifle+bullet system does not change.

- Force on b is equal and opposite to force on R (3rd law), and the two forces act for the same time same impulse delivered to each, but in opposite direction same change in momentum for R as for b, but in the opposite direction i.e. the momentum changes for the system cancel to zero. Momentum is conserved.



$M\,v = -\,m\,V$

Both the rifle and the bullet gain considerable momentum, but the (rifle+bullet) system experiences zero momentum change.

Note the importance of direction (as well as size), when considering momentum.

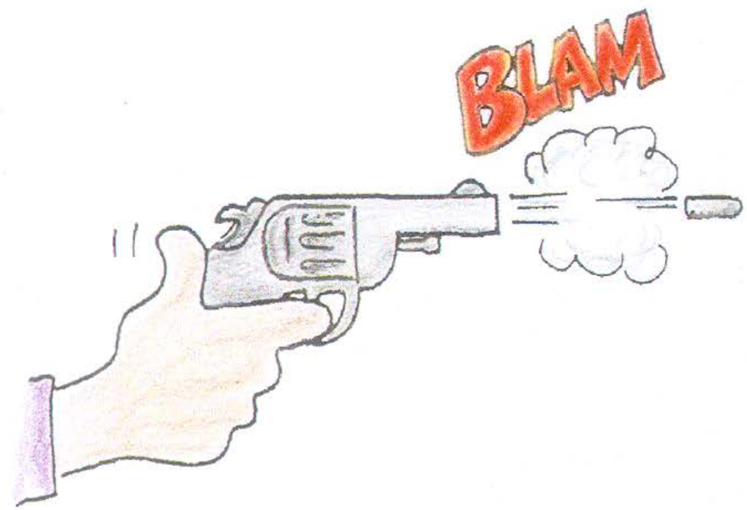
mom 1: cannon redux

Strictly speaking, when a gun is fired, compared with the momentum of the recoiling gun, the opposite momentum of the bullet is

a) less.

b) more.

c) the same.



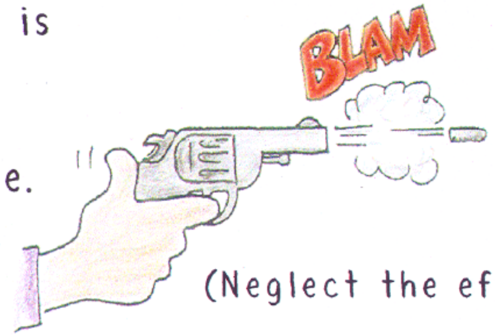
(Neglect the effect of the hand.)

thanx to David G. Willey

Hewitt Drew it!

Strictly speaking, when a gun is fired, compared with the momentum of the recoiling gun, the opposite momentum of the bullet is

a) less.
b) more.
c) the same.

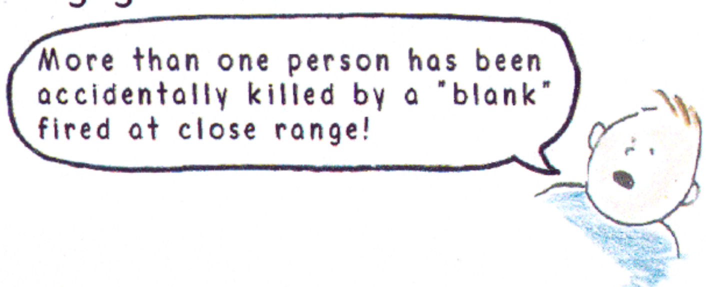


(Neglect the effect of the hand.)

Answer: a

Why? Because more than just a bullet comes out of the barrel when a gun is fired. The gas, formed when the powder in the cartridge burns, pushes the bullet along the barrel and this gas too has appreciable mass and exits at high speed. More than negligible momentum is given to the gases. So,

momentum of recoiling gun = momentum of bullet + momentum of gases



of system of
colliding objects

Collisions

- Momentum is conserved during a collision, because all forces acting in collision are internal:

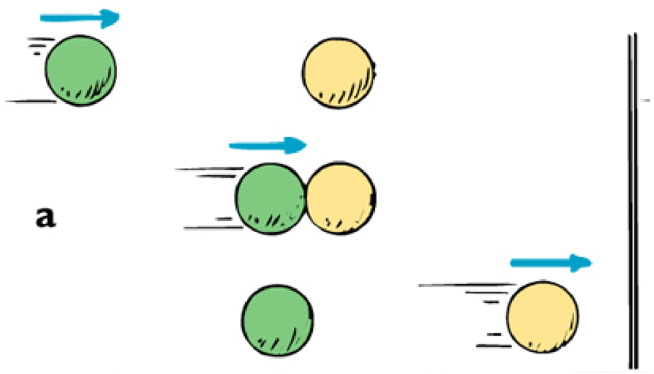
Net momentum before collision = net momentum after collision

Momentum is redistributed among the participants of the collision.

Example: Two equal-mass balls colliding:

a) The moving ball comes to rest, the other moves off with the speed of the colliding ball.

$p_i = p_1$



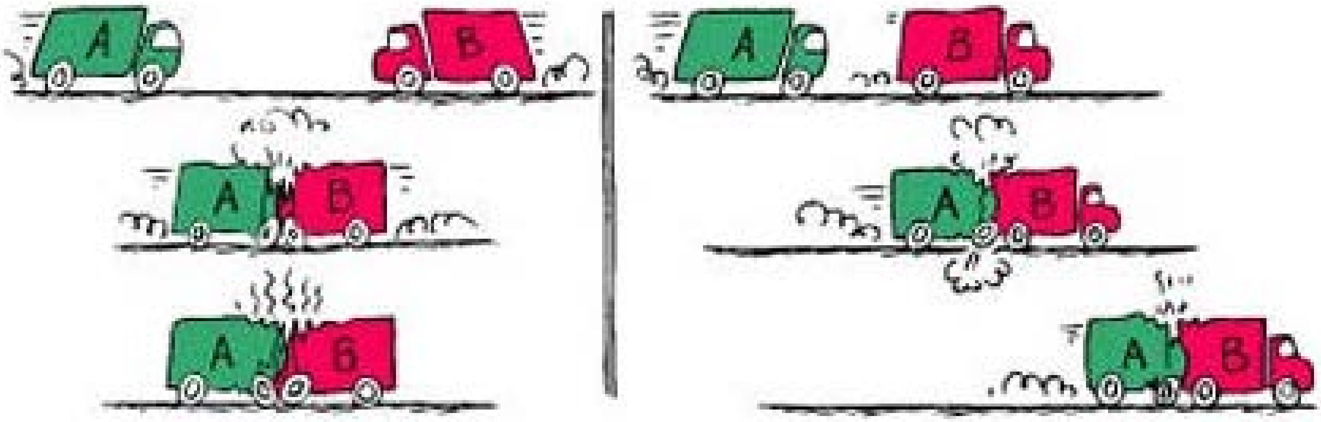
$p_f = p_2 = p_i$

mom 2 1D elastic collision

These are both **elastic collisions** – no lasting deformation or heat or sound…

Many collisions are **inelastic** - where heat and/or sound is generated, and/or objects deform. Even so, **momentum is still conserved** .

Eg.



Note that net momentum before = net momentum after (always in collision, whether elastic or inelastic)

mom 3 inelastic collision

Question

A garbage truck and a mini car have a head-on collision.

a) Which vehicle experiences the greater force of impact?

Both same (action-reaction, 3rd law)

b) Which experiences the greater impulse?

Both same (same force over same time interval)

c) Which experiences the greater momentum change?

Both same (momentum of system conserved, so momentum change of truck is equal and opposite to the momentum change of the car)

d) Which experiences the greater acceleration?

The car (smaller mass)

Question continued

e) Say the garbage truck weighs 15 000-kg, and the mini car weighs 1000 kg. Let's say the truck is initially moving at 30 km/h and the car is at 60 km/h.

If the two stick together after the collision, then what is their speed after the head-on collision?

Momentum conservation means:

mom. of truck before + mom. of car before = mom of (car+truck) after

i.e. $m_t\, v_t - m_c\, v_c = (m_t + m_c)\, v$ *(- on left because opp dir)*

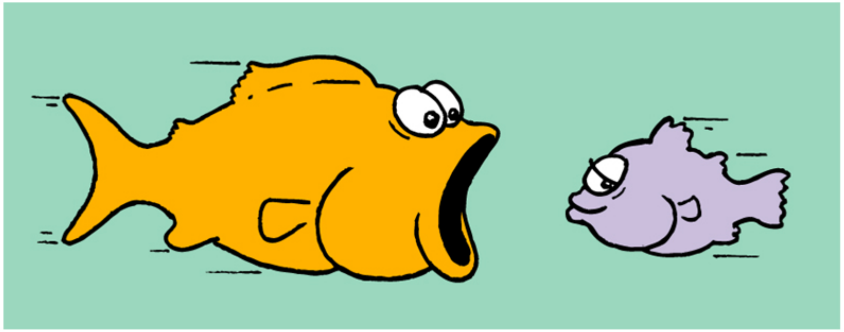
(15000)(30) - (1000)(60) = (16000) v

So, v = 24.375 km/h

Note that they do eventually come to rest because of friction on the road – an external force. Since the impact time is relatively short, we can ignore this external force during the collision since it is much smaller than the collisional impact force. Hence we assume momentum is conserved in the collision.

Another Question

The orange fish has mass 4-kg, and the purple one has mass 1-kg.



a) If the orange fish is swimming at 2 m/s towards the purple fish at rest, what is the speed of orange fish after he swallows him? Neglect water resistance.

Net momentum before = net momentum afterwards

(4 kg)(2 m/s) + (1 kg)(0) = ((4+1)kg) v

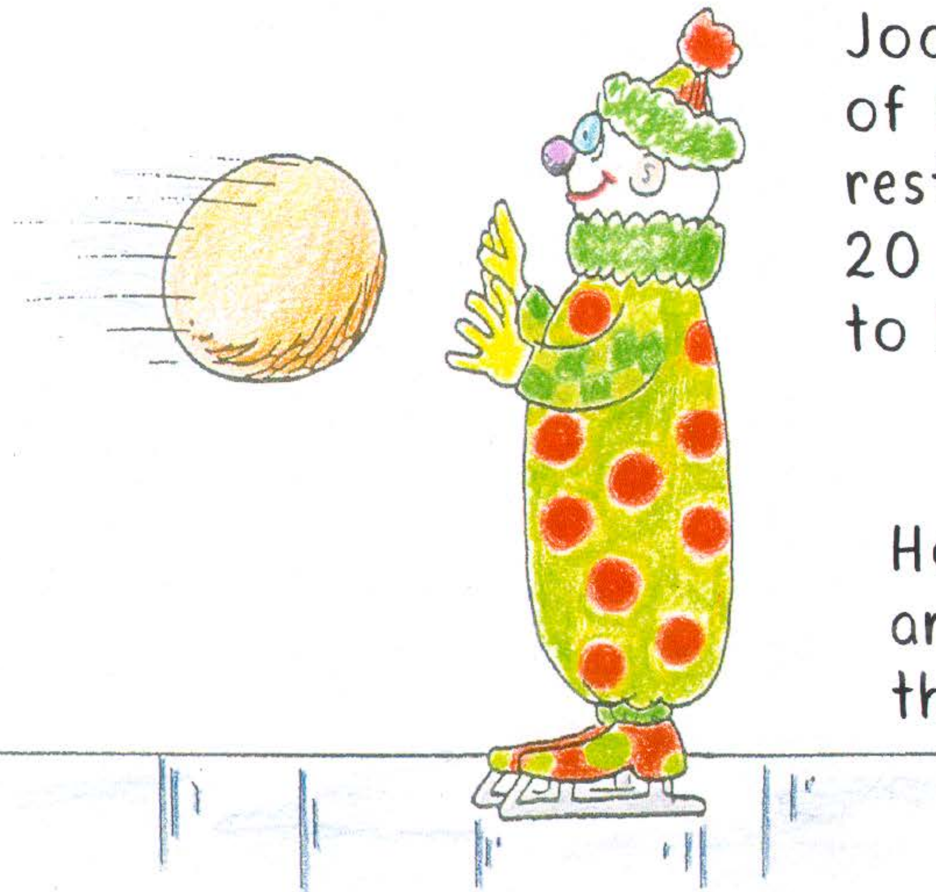
8 kg m/s = (5 kg) v . So v = 8/5 m/s = <u>1.6 m/s</u>

b) If instead the purple fish sees the orange fish coming, and swims away at 1m/s, then what is the speed of the orange fish, after he catches up and swallows him?

Net momentum before = net momentum afterwards

(4 kg)(2 m/s) + (1kg)(1m/s) = ((4+1)kg) v

9 kg m/s = (5 kg) v . So v = 9/5 m/s = <u>1.8 m/s</u>



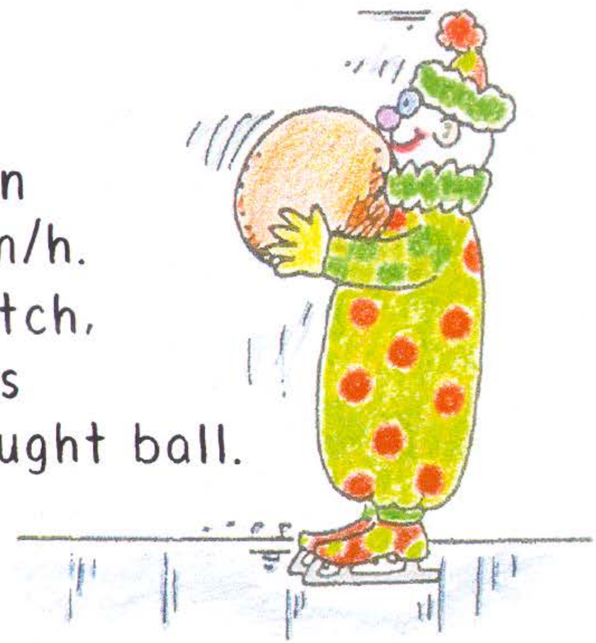
Jocko, who has a mass of 60 kg and stands at rest on ice, catches a 20 kg ball that is thrown to him at 10 km/h.

How fast does Jocko and the ball move across the ice?

Hewitt Drew it!

Jocko, who has a mass of 60 kg and stands at rest on ice, catches a 20 kg ball that is thrown to him at 10 km/h.

How fast does Jocko and the ball move across the ice?



Answer: 2.5 km/h

The momentum before the catch is all in the ball, 20 kg × 10 km/h = 200 kg·km/h. This is also the momentum after the catch, where the moving mass is 80 kg. That's 60 kg for Jocko and 20 kg for the caught ball.

$$(60 + 20)\ \text{kg} \times v = 200\ \text{kg·km/h}$$

$$v = \frac{200\ \text{kg·km/h}}{80\ \text{kg}} = 2.5\ \text{km/h}$$

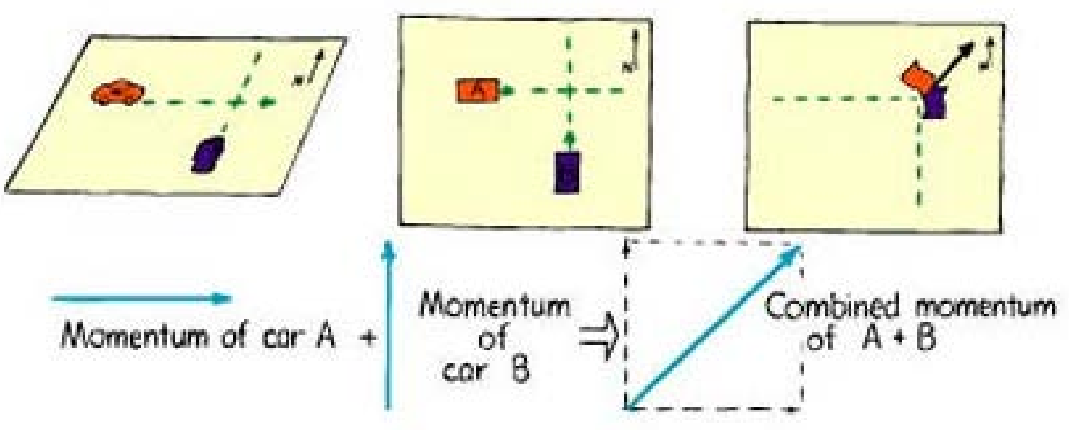
Hewitt Drew it!

Collisions in more than 1 dimension

The net momentum in any direction still remains unchanged. Need to use parallelogram rule to figure out net momentum vector.

We'll just look at some simpler cases

Eg. Car A traveling down Lexington Ave at 40 mph, crashes with Car B, with same mass as Car A, traveling down 68th St also at 40 mph, and stick together. Which direction do they move off in and at what speed (initially)?

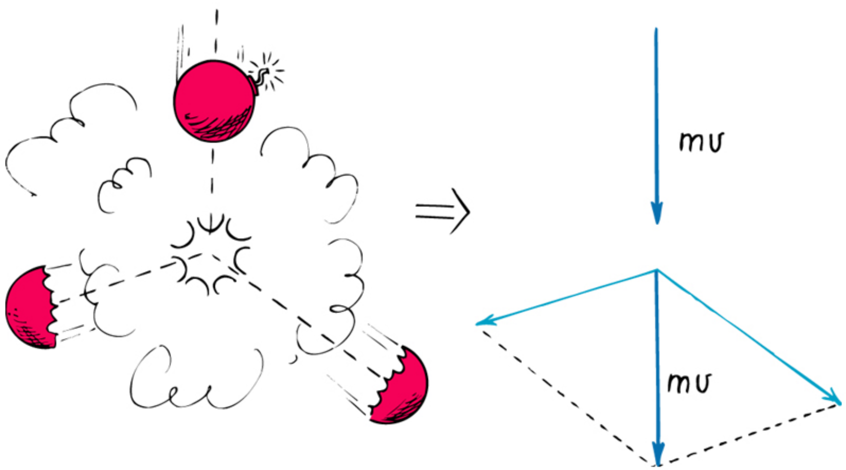


The diagonal of square has length $\sqrt{2}$ times length of one side. So here, the resultant speed is 40 $\sqrt{2}$ mph = 56.6 mph.

Direction is north-east in picture shown, i.e. at 45 degrees to both Lexington and 68th St.

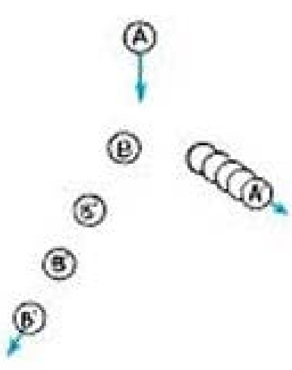
mom 4, 2D collision

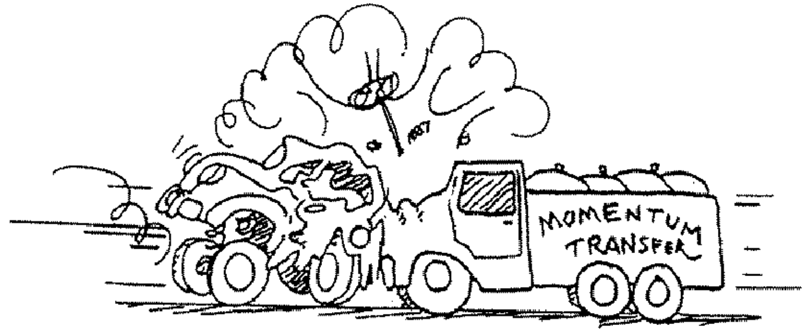
Eg. Firecracker exploding as it is falling, (or a radioactive nucleus breaking up..)



Momenta of final fragments add to give net momentum equal to the initial.

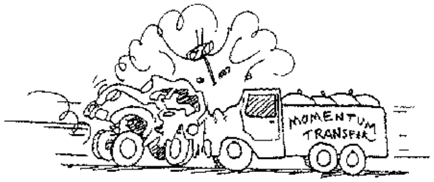
Eg. Billiard balls – ball A strikes B which was initially at rest. Parallelogram with A' and B gives original momentum of A.





Whenever an interaction occurs in a system, forces occur in equal and opposite pairs. Which of the following do *not* always occur in equal and opposite pairs?

1. Impulses.
2. Accelerations.
3. Momentum changes.
4. All of these occur in equal and opposite pairs.
5. None of these do.



Whenever an interaction occurs in a system, forces occur in equal and opposite pairs. Which of the following do *not* always occur in equal and opposite pairs?

1. Impulses.
2. **Accelerations.** ✔
3. Momentum changes.
4. All of these occur in equal and opposite pairs.
5. None of these do.

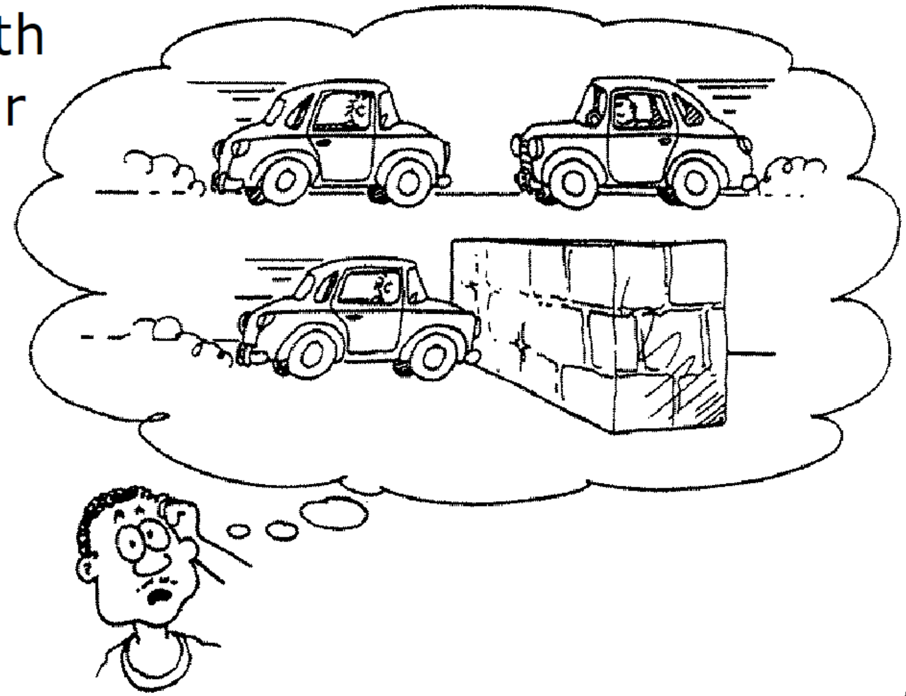
Answer: 2

Because time for each interaction part is the same, impulses and momentum changes also occur in equal and opposite pairs. But not necessarily accelerations, because the masses of the interaction may differ. Consider equal and opposite forces acting on masses of different magnitude.

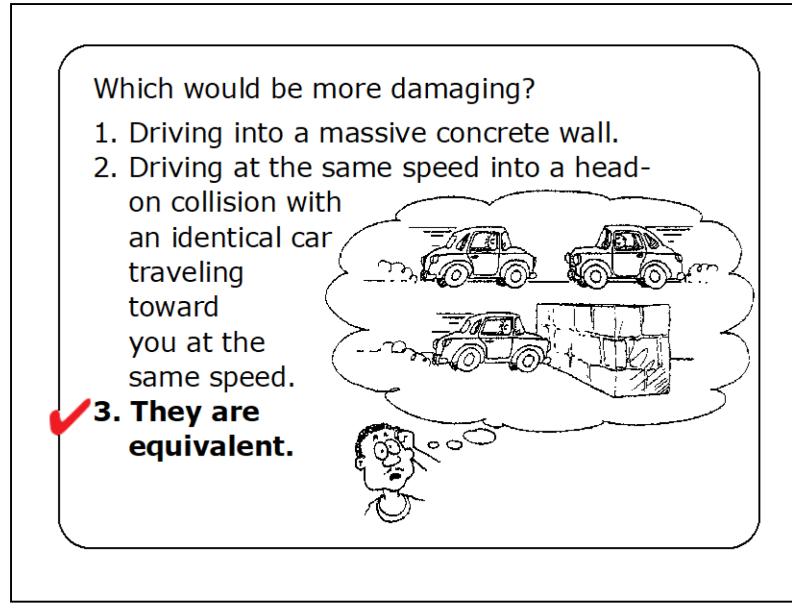
$$\frac{F}{\Huge m} = a \qquad \frac{F}{m} = {\Huge a}$$

Which would be more damaging?

1. Driving into a massive concrete wall.
2. Driving at the same speed into a head-on collision with an identical car traveling toward you at the same speed.
3. They are equivalent.



Note! In answering this, assume the collision time is the same in each case (may be unrealistic), and also assume you are concerned only with the damage done to *your own* car.



Answer: 3

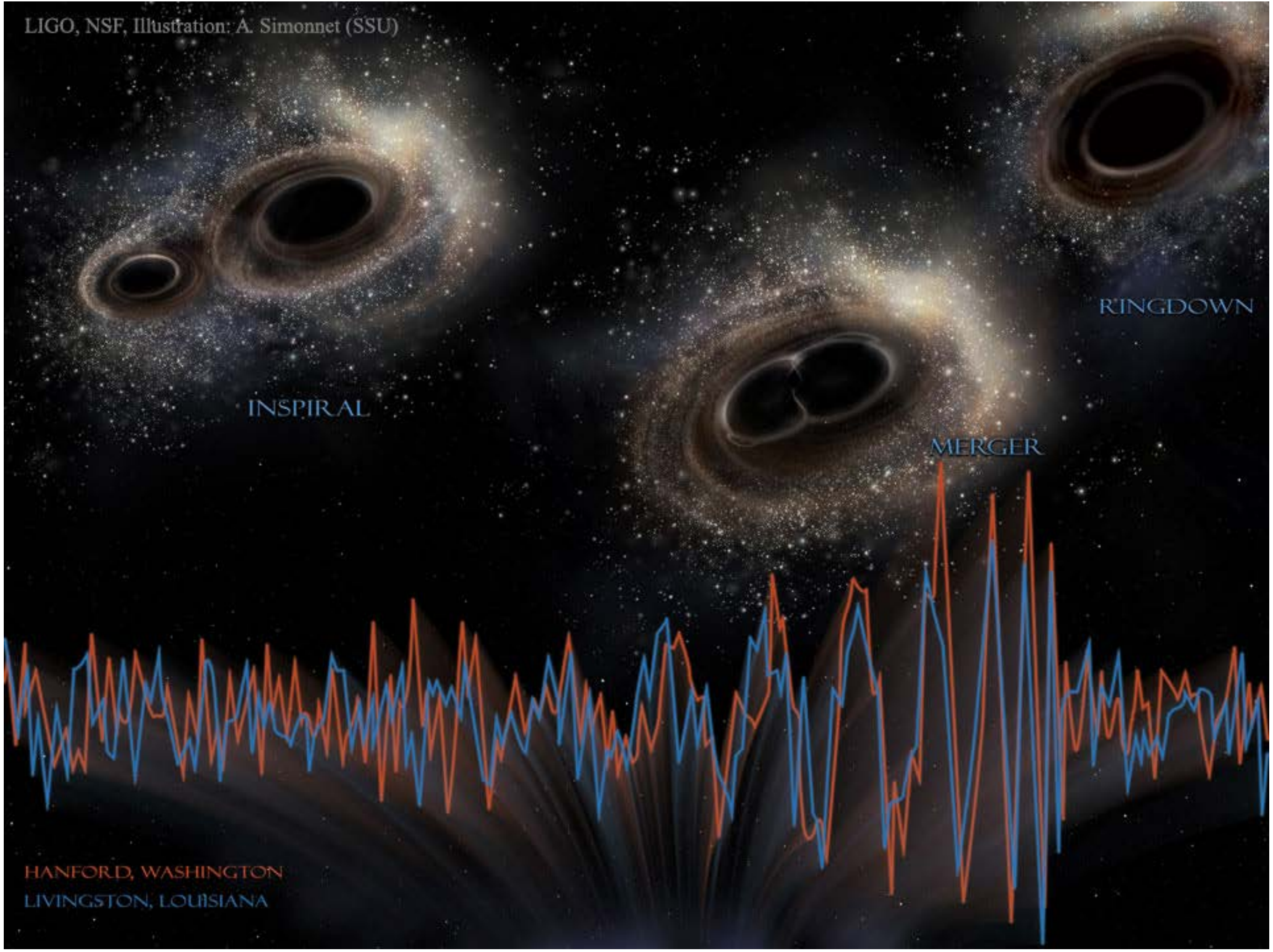
Your car decelerates to a dead stop either way. The dead stop is easy to see when hitting the wall, and a little thought will show the same is true when hitting the car. If the oncoming car were traveling more slowly, with less momentum, you'd keep going after the collision with more "give," and less damage (to you). But if the oncoming car had more momentum than you, it would keep going and you'd snap into a sudden reverse with greater damage. Identical cars at equal speeds means equal momenta—zero before, zero after collision.

A pair of spiral galaxies collide and merge to form one larger elliptical galaxy. Astronomers assume that the momentum of the new elliptical galaxy is

a) equal to the sum of the momenta of two spiral galaxies.
b) equal to the difference in momenta of the two spiral galaxies.
c) the same as the momentum of the more massive spiral galaxy.
d) zero.

A pair of spiral galaxies collide and merge to form one larger elliptical galaxy. Astronomers assume that the momentum of the new elliptical galaxy is

a) **equal to the sum of the momenta of two spiral galaxies.**
b) equal to the difference in momenta of the two spiral galaxies.
c) the same as the momentum of the more massive spiral galaxy.
d) zero.



LIGO Detects Gravitational Waves from Merging Black Holes
Illustration Credit: LIGO, NSF, Aurore Simonnet (Sonoma State U.)

https://youtu.be/4GbWfNHtHRg

Conservation Laws are Fundamental to Physics

Linear Momentum

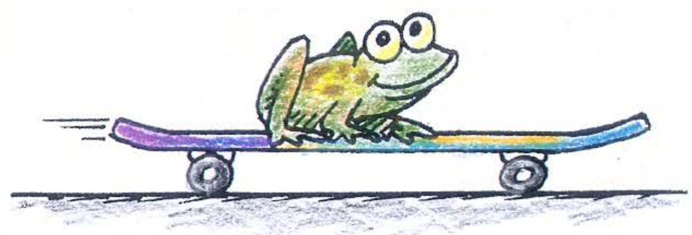
Energy

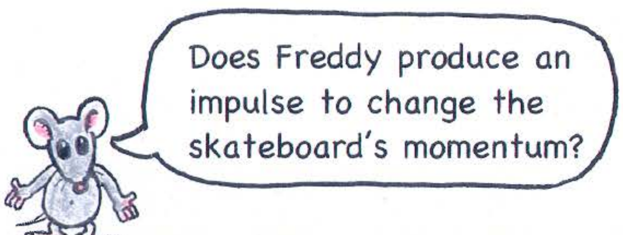
Angular Momentum

NEXT-TIME QUESTION

Freddy the frog rides on a skateboard on a horizontal surface. Freddy then jumps vertically relative to the board and catches onto a tree limb above. Immediately after the jump, the speed of the skateboard

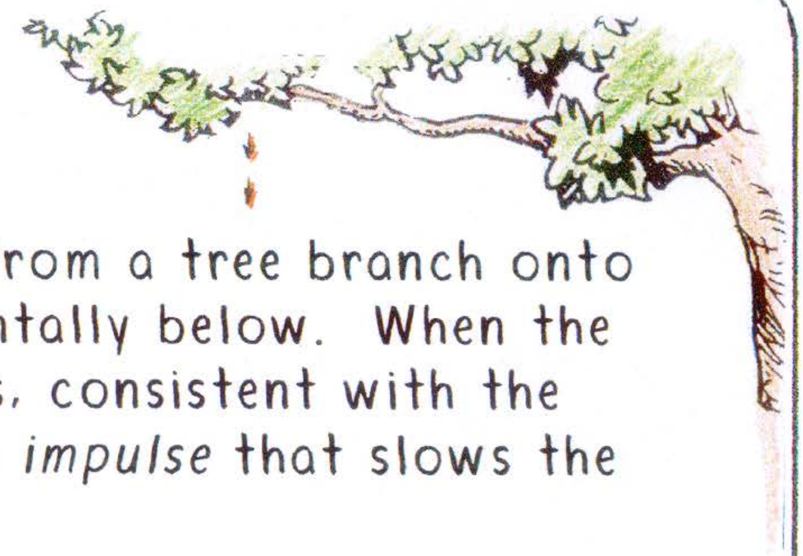
A. increases.
B. decreases.
C. remains unchanged.





Hewitt Drew it!

NEXT-TIME QUESTION



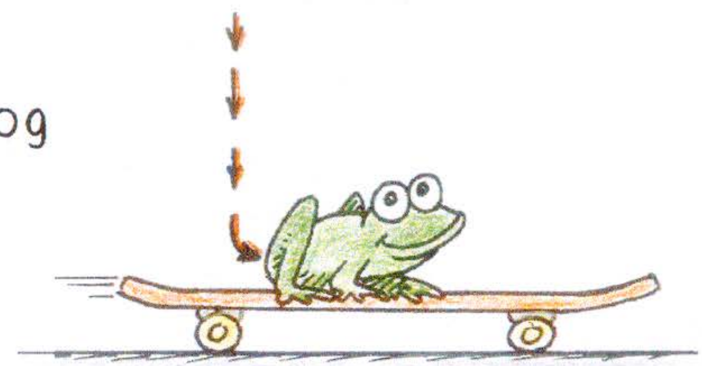
A massive frog drops vertically from a tree branch onto a skateboard that moves horizontally below. When the frog lands, the skateboard slows, consistent with the conservation of momentum. The *impulse* that slows the skateboard is

a) the friction force of the frog's feet acting backward on the skateboard × time during which the speed changes.

b) equal and opposite to the impulse that brings the frog up to speed.

c) Both of these.

d) Neither of these.



Hewitt Drew it!