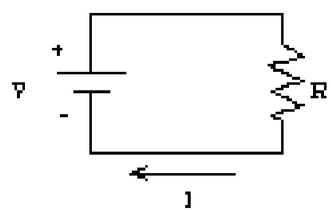
■ 7.Electrical Circuits and Kirchhoff's Rules

Electrical circuits involving batteries and resistors can be treated using a method of analysis developed by Kirchoff. There are just two Kirchhoff's rules: the **loop** rule and **node** rule.

□ An example of a loop--Ohm's law:

A **loop** is a closed electrical path. A simple example of a loop with a battery V and resistor R is indicated below:



The (ideal) wires have no resistance so the battery voltage is the same as the voltage across the resistor R (measured in **ohms**). Ohm's law

$$V = I\,R \tag{20.1}$$

is an experimental law which connects the electrical potential (measured in **volts**) applied across the resistor with the current I (measured in **amps**) through the resistor. Notice the current I supplied by the battery is the same as the current in the resistor. It is usual to write eqt. (20.1) in the form

$$V - I\,R = 0 \tag{20.2}$$

and refer to **−IR** as the voltage **drop** across the resistor. The voltage is lower at the bottom of the resistor (where the current leaves) than at the top (where the current enters).
Kirchhoff's loop rule for this simple circuit is equivalent to eqt. (20.2) as will be seen below.

Typically, the numerical values of the voltages and resistance are given and you are asked to calculate the currents in the circuit. For example, if V=2.0 volts and R=4 ohms in the above circuit then

```
V=2.0; R=4.;
Print["i = ", i=V/R]
```

```
i = 0.5
```

so a current of 0.5 amps is the result.

Kirchhoff's Loop Rule: Conservation of Electrical Potential

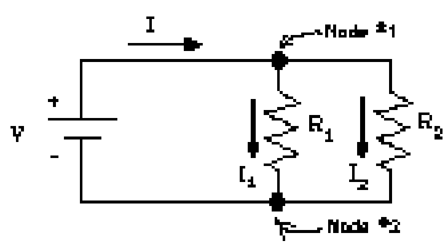
The Kirchhoff's loop rule states the sum of the potential differences ΔV about an electrical loop is zero, or

$$\sum_{\text{loop}} \Delta V = 0 \quad \text{Loop Equation} \tag{20.3}$$

ΔV is the voltage across an electrical component like a battery or resistor. The **source** of electrical potential (the **battery**) is included in eqt. (20.3) with $\Delta V = V$ the battery voltage. The voltage across a **resistor** is included in eqt. (20.3) with $\Delta V = -IR$ (Ohm's law). Notice the minus sign as more will be said about this in a moment.

A More Complicated Circuit:

A circuit with two resistors R_1 and R_2 in parallel with a battery V appears below



This circuit has two places (called **nodes**) where three or more wires come together. The current I into node # 1 is equal to the sum of the currents leaving the nodes (I_1 and I_2) since current is conserved. Thus

$$I = I_1 + I_2 \tag{20.4}$$

Notice that I_1 is the current in resistor R_1 , I_2 is the current in R_2 , and I is the current in the battery V. Also notice that the same equation results from looking at the currents at node #2.

Kirchhoff's Node Rule: Conservation of Current

The Kirchhoff **node rule** is

$$\sum I_{in} = \sum I_{out} \quad \text{Node Equation} \tag{20.5}$$

This is a generalization of eqt. (20.4) to the case where there are more wires at the node.

◻ Some Sign Conventions (+ or −) for the Kirchhoff Loop Rule:

The second circuit above has three loops or closed electrical circuits. One loop includes the battery V and resistor R_1, a second loop involves the battery V and resistor R_2, and the third loop involves resistors R_1 and R_2. There are two preliminary things to keep in mind:

(1)Assign a symbol for an unknown **current** in each resistor and **guess** at the direction of this current. (If you guess wrong about the direction of the current do not worry. At the end of a calculation, if you get a negative value for the current, then you just guessed wrong about the direction of the physical current.) Assign a current to each battery as well.
(2)Always travel around the electrical loops in the **clockwise** direction when considering whether a given potential is **positive** or **negative**.

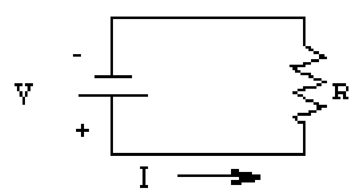
SIGN RULE FOR VOLTAGE ACROSS A RESISTOR:

If the current in the resistor is in the **same** direction as the direction you travel about the loop (the **clockwise** direction), then the voltage drop is – IR across the resistor. (If the current in the resistor is in the direction **opposite** to the direction you travel about the loop (the clockwise direction), then the voltage drop + IR across the resistor is **positive**.)

SIGN RULE FOR VOLTAGE ACROSS A BATTERY:

If you encounter a battery and you go from the negative to the positive terminals, then the voltage produced by the battery is considered **positive.** (If you encounter a battery and you go from the positive to the negative terminals, then the voltage produced by the battery is considered **negative**.)

The above conventions might seem a bit complicated but after application to a few simple examples they will appear more simple and reasonable. Consider the first example of a single loop and reverse the direction of the battery so you have

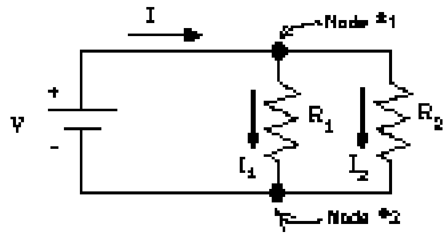


The current in the battery leaves the positive terminal by the convention above and this is the same current as in the resistor. The voltage due to the battery (as you travel around the loop in the clockwise direction) is –V since you enter by the + terminal of the battery and you exit by the – terminal. The voltage drop across the resistor is + IR since the current I is in the opposite direction to the clockwise direction of travel. Application of the loop equation (20.3) yields

$$-V + IR = 0 \qquad (20.6)$$

Solving eqt. (20.6) with V = 2.0 and R = 4, you get the same magnitude for the current I=0.5 amps as before with eqt. (20.2). The direction of the current I is different in this example since the battery was reversed.

The circuit below involving two resistors is a more complicated example.



Eqt. (20.4) the node equation was already written down. The loop equation for the battery and resistor R_1 is

$$V - I_1 R_1 = 0 \tag{20.7}$$

while the loop equation for the battery and the resistor R_2 appears

$$V - I_2 R_2 = 0 \tag{20.8}$$

The loop equation involving the resistors R_1 and R_2 is

$$I_1 R_1 - I_2 R_2 = 0 \tag{20.9}$$

This last equation does not include any additional information since eqt. (20.9) can be obtained by adding eqt. (20.7) and eqt. (20.8).

The loop equations (20.7) and (20.8) together with the node equation (20.4) are enough to calculate the curents I, I_1, and I_2 . Suppose V= 2.0 volts, R_1=1000 ohms, and R_2=2000 ohms. Solving eqt. (20.7) for I_1 yields

$$I_1 = \frac{V}{R_1} = \frac{2.0\ \mathrm{V}}{1000\ \Omega} \tag{20.10}$$

and solving eqt. (20.8) for I_2 yields

$$I_2 = \frac{V}{R_2} = \frac{2.0\ \mathrm{V}}{2000\ \Omega} \tag{20.11}$$

Finally, utilization of I_1 and I_2 in node eqt. (20.4) yields

$$I = \frac{2.0\ \mathrm{V}}{1000\ \Omega} + \frac{2.0\ \mathrm{V}}{2000\ \Omega} \tag{20.12}$$

Mathematica gives the following numerical values for the currents

```
R1=1000.;R2=2000.; V=2.0;
Print["i1 =", i1=V/R1]
Print["i2 =", i2=V/R2]
Print["i3 =", i3=V/R1+V/R2]
```

```
i1 =0.002
i2 =0.001
i3 =0.003
```

There are **two separate steps** needed to solve for **currents** in circuits:

(i)The first step involves application of the **Kirchhoff method** to get the node and loop equations.

(ii)The second step is mathematical and involves the **algebraic solution** of the node and loop equations for the currents.

The *Mathematica* function **Solve** can be used to perform this algebra so that circuit problems are much more easily solved.

□ The Solve[ ] Function:

The function **Solve** has two **arguments** separated by a **comma**. The first argument is the **list of equations** you wish to solve. The second argument is the set of **variables `{I1,I2,Ib}`** for which you wish to solve. (Use **Ib** instead of **I** since *Mathematica* reserves **I** for the complex number square root of minus one.) The solution to the circuit above is obtained as follows:

```
R1=1000.;R2=2000.;V=2.0;
Solve[{V==I1*R1,V==I2*R2,Ib==I1+I2},{I1,I2,Ib}]
```

```
{{I1 -> 0.002, I2 -> 0.001, Ib -> 0.003}}
```

Notice the values of the resistors and the battery voltage were input as before. The first argument of the **Solve[ ]** function involves the list of three equations

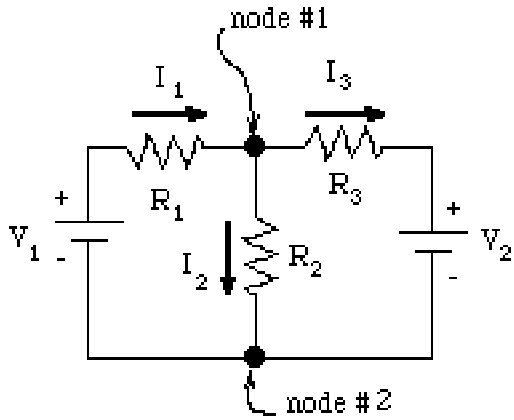
```
{V==I1*R1,V==I2*R2,Ib==I1+I2}
```

The items of this list (the equations) are separated by **commas** and the entire list is enclosed in curly brackets **{ }**. The **double** equal signs == in the equations is understood by *Mathematica* in the same sense as an equal sign is used in algebra; namely, what is on the right of == is the same as what is on the left and vice versa.

Recall that a **single** equal = is understood by *Mathematica* as **take what is on the right side of = and assign it to the left side of the equal sign**. A **single** equal = in *Mathematica* has a different meaning than in algebra and an arrow ⇐ would be better.

A Final Example:

Apply Kirchhoff's rules to the circuit below:



where V_1=15 V, V_2=5 V, R_1=60Ω, R_2=40Ω, and R_3=30Ω. There are two (independent) loop equations:

$$V_1 - I_1 R_1 - I_2 R_2 = 0 \tag{20.13}$$

$$-V_2 - I_3 R_3 + I_2 R_2 = 0 \tag{20.14}$$

and one (independent) node equation

$$I_1 = I_2 + I_3 \tag{20.15}$$

The solution to the above set of equations appears

```
Clear[I1,I2,I3];
V1=15.;V2=5.;R1=60.;R2=40.;R3=30.;
Solve[{V1-I1*R1-I2*R2==0.,
       -V2-I3*R3+I2*R2==0.,
        I1==I2+I3},
       {I1,I2,I3}]
```

```
{{I1 -> 0.157407, I2 -> 0.138889, I3 -> 0.0185185}}
```