

Assignment #4

45. Two charged concentric spherical shells have radii $R_1 = 10 \text{ cm}$ and $R_2 = 15.0 \text{ cm}$. The charge on the inner shell is $Q_1 = 4.00 \times 10^{-8} \text{ C}$ and the charge on the outer shell is $Q_2 = 2.0 \times 10^{-8} \text{ C}$. Find the electric field at (a) $r = 12.0 \text{ cm}$ and (b) $r = 20.0 \text{ cm}$.

Solution (a) When $r = 12.0 \text{ cm}$, we have $R_1 < r < R_2$.

In this case, use a Gaussian surface as a sphere of radius r . Charge within the Gaussian surface is Q_1 .

So, by Gauss' law, the electric field at distance r is

given by
$$(4\pi r^2) E_1 = \frac{Q_1}{\epsilon_0}$$

$$\rightarrow E_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r^2} = 2.5 \times 10^4 \text{ N/C}$$

(b) When $r = 20 \text{ cm}$, again use a Gaussian surface which is a sphere of radius r . Total charge enclosed by the Gaussian surface is $Q_1 + Q_2$.

By Gauss' theorem, the electric field is given as the solution of

$$(4\pi r^2) E_2 = \frac{Q_1 + Q_2}{\epsilon_0}$$

$$\begin{aligned} \rightarrow E_2 &= \frac{1}{4\pi\epsilon_0} \frac{Q_1 + Q_2}{r^2} \\ &= 1.35 \times 10^4 \text{ N/C} \end{aligned}$$

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46. Assume that a ball of charged particles has a uniformly distributed negative charge density except for a narrow radial tunnel through its center from the surface on one side to the surface on the opposite side. Also, assume we can position a proton anywhere along the tunnel or outside the ball. Let F_p be the magnitude of the electrostatic force on the proton when it is located

when it is located at the ball's surface, at radius R .

As a multiple of R , how far from the surface is there a point where the force magnitude is $0.5F_P$, if we move the proton

(a) away from the ball?

and (b) into the tunnel?

Solution The field at the proton's location (not caused by the proton itself) is E_P . The charge on the proton is e . The charge on the ball is q . Therefore, if $r > R$, the force on the proton caused by the ball is

$$F = eE = e \left(\frac{q}{4\pi\epsilon_0 r^2} \right) = \frac{eq}{4\pi\epsilon_0 r^2}$$

where r is measured from the center of the ball to the point.

If $r = R$, this expression yields

$$F_R = \frac{eQ}{4\pi\epsilon_0 R^2}$$

(b) ^{Now} If we require

$$F = \frac{1}{2} F_R$$

↑
inside

$$e \left(\frac{Q}{4\pi\epsilon_0 R^3} r^3 \right) = \frac{1}{2} \left(\frac{eQ}{4\pi\epsilon_0 R^2} \right)$$

$$\rightarrow r = \frac{1}{2} R$$

$$(a) \quad e \left(\frac{Q}{4\pi\epsilon_0} \frac{1}{r^2} \right) = \frac{1}{2} \left(\frac{eQ}{4\pi\epsilon_0 R^2} \right)$$

$$\rightarrow r = \sqrt{2} R$$

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47. An unknown charge sits on a conducting solid sphere of radius $R = 10 \text{ cm}$. If the electric field $r = 15 \text{ cm}$ from the center of the sphere has magnitude $E = 3.0 \times 10^3 \text{ N/C}$ and is directed radially inward, what is the net charge on the sphere?

Solution

The unknown charge is distributed uniformly over the surface of the sphere.

The electric field produced by the unknown charge at points outside the sphere is like the field of a point particle with charge equal to the net charge on the surface of the sphere.

$$\begin{aligned} \rightarrow |q| &= 4\pi\epsilon_0 r^2 E \\ &= \frac{(0.15 \text{ m})^2 (3.0 \times 10^3 \text{ N/C})}{8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} \\ &= 7.5 \times 10^{-9} \text{ C} \end{aligned}$$

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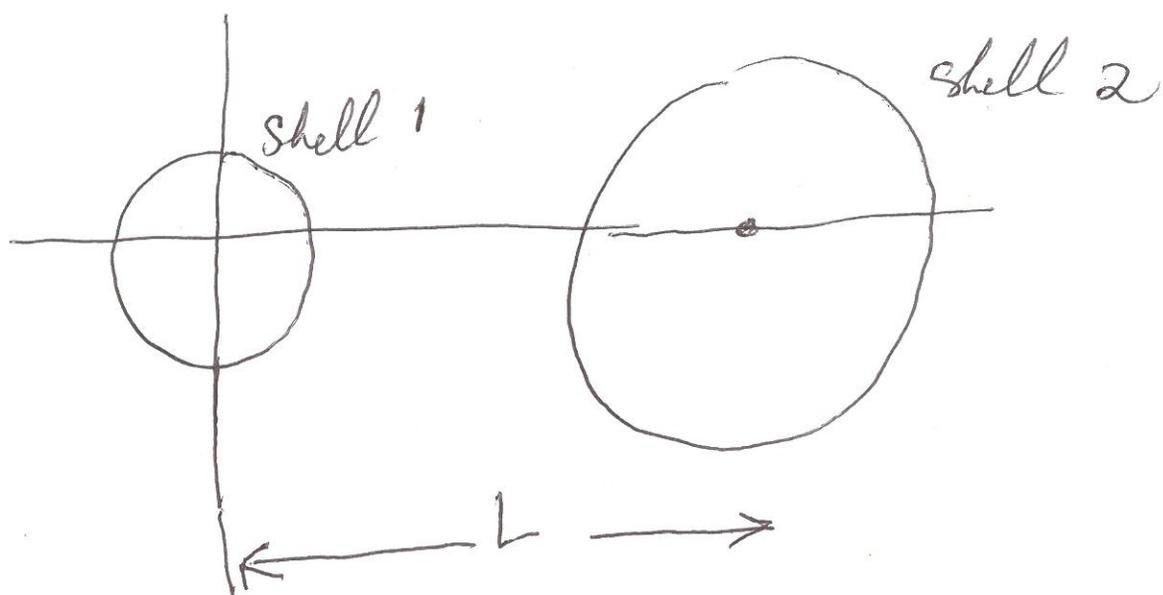


Figure shows two non-conducting spherical shells fixed in place on the x-axis. Shell #1 has uniform surface charge density $\sigma_1 = +4.0 \times 10^{-6} \text{ C/m}^2$ on its outer surface and radius $R_1 = 0.5 \text{ cm}$. Shell 2 has uniform surface charge density $\sigma_2 = -2.0 \times 10^{-6} \text{ C/m}^2$ on its outer surface and radius $R_2 = 2.0 \text{ cm}$. Their centers are separated by $L = 6.0 \text{ cm}$. Other than $x = \infty$, where on the x-axis is the net electric field equal to zero?

Solution

The point where the net electric field is zero cannot be inside the shells since there is only one field contributing. It cannot be between the shells since the charges on the shells have opposite signs.

Shell 2 has greater charge $q_2 = |\sigma_2| A_2$ than shell 1 with $q_1 = |\sigma_1| A_1$. So, the point where the net electric field is zero is to the left of shell #1 at a distance $X > R_1$ from its center.

This gives $E_1 = E_2$

$$\frac{|q_1|}{4\pi\epsilon_0 X^2} = \frac{|q_2|}{4\pi\epsilon_0 (X+L)^2}$$

$$\rightarrow \frac{\sigma_1 A_1}{4\pi\epsilon_0 X^2} = \frac{\sigma_2 A_2}{4\pi\epsilon_0 (X+L)^2}$$

Using $A = 4\pi R^2$ for the surface area
of a sphere gives

$$X = -L \left(\frac{R_1}{R_2} \right) \frac{1}{\sqrt{10_2/10_1 - 1}} \approx 3.3 \text{ cm}$$