

## Assignment #6

### Chapter 25

#40 An air-filled parallel plate capacitor has a capacitance of  $C_1 = 1.3 \text{ pF}$ . The separation of the plates is doubled and wax is inserted between them. The new capacitance is  $C_2 = 2.6 \text{ pF}$ . Find the dielectric constant  $\kappa$  of the wax.

Solution

$$C_1 = \frac{\epsilon_0 A}{d}, \quad C_2 = \frac{\epsilon_0 A \cdot \kappa}{2d}$$

$$\rightarrow \frac{C_1}{C_2} = \frac{\epsilon_0 A/d}{\kappa \epsilon_0 A/2d} = \frac{2}{\kappa}$$

$$\rightarrow \kappa = 2 \frac{C_2}{C_1} = 2 \frac{(2.6 \text{ pF})}{(1.3 \text{ pF})} = 2 \times 2 = 4$$

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#47 A certain substance has a dielectric constant of  $\epsilon_r = 2.8$  and a dielectric strength of  $E_c = 18 \text{ MV/m}$ . If it is used as the dielectric material in a parallel plate capacitor, what minimum area should the plates of the capacitor have to obtain a capacitance of  $C = 7.0 \times 10^{-2} \mu\text{F}$  and to ensure the capacitor will be able to withstand a potential difference of  $V = 4.0 \text{ kV}$ ?

Solution

$$E_c \geq V/d \Rightarrow d \geq V/E_c$$

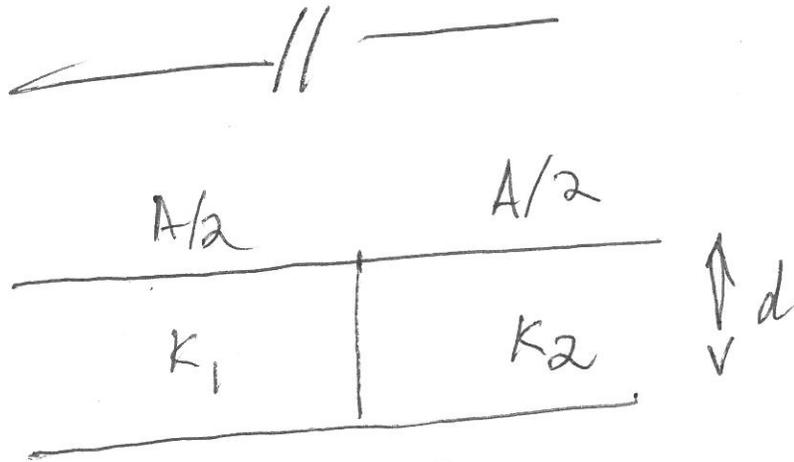
$$C = \epsilon_r \epsilon_0 A/d$$

$$\rightarrow \frac{\epsilon_r \epsilon_0 A}{C} \geq V/E_c$$

$$\rightarrow A \geq \frac{VC}{E_c \epsilon_r \epsilon_0}$$

$$\rightarrow A = \frac{(7.0 \times 10^{-8} \text{ F}) (4.0 \times 10^3 \text{ V})}{(2.8) (8.85 \times 10^{-12} \text{ F/m}) (18 \times 10^6 \text{ V/m})}$$

$$= 0.63 \text{ m}^2$$



#48

$$A = 5.56 \text{ cm}^2$$

$$d = 5.56 \text{ mm}$$

$$k_1 = 7.0 ; k_2 = 12.0$$

Soln Have two capacitors in parallel. Equivalent capacitance is  $C = C_1 + C_2$

where  $C_1 = \frac{k_1 \epsilon_0 A/2}{d}$

$$C_2 = \frac{k_2 \epsilon_0 A/2}{d}$$

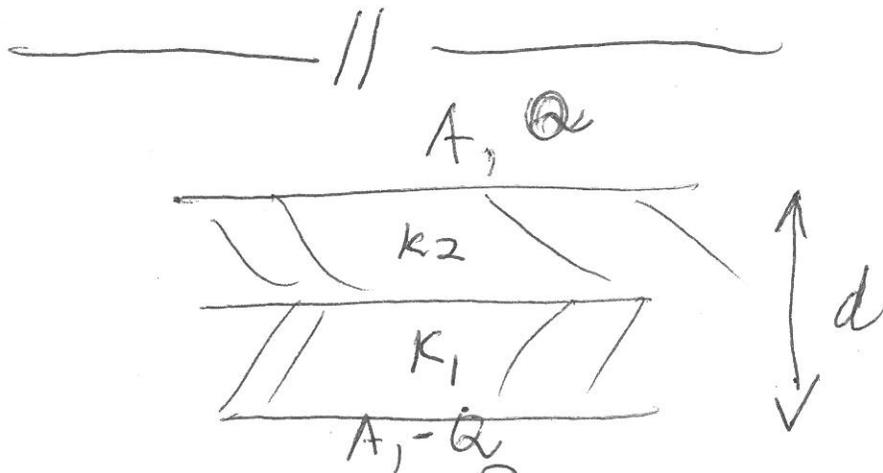
Q20,

$$C = \frac{\epsilon_0 A}{d} (k_1 + k_2)$$

$$= \frac{\epsilon_0 A}{d} \left( \frac{k_1 + k_2}{2} \right)$$

$$= 8.41 \times 10^{-12} \text{ F}$$

#49



$$A = 7.89 \text{ cm}^2$$

$$d = 4.62 \text{ mm}$$

$$k_1 = 11; k_2 = 12$$

Solution

Have two capacitors in series

$$C_1 = k_1 \epsilon_0 A / d_1; C_2 = k_2 \epsilon_0 A / d_2$$

$$\rightarrow \frac{1}{C} = \frac{d_1}{k_1 \epsilon_0 A} + \frac{d_2}{k_2 \epsilon_0 A}$$

$$\text{Taking } d_1 = d_2 = d/2 \Rightarrow$$

$$\frac{1}{C} = \frac{d/2}{\epsilon_0 A} \left( \frac{1}{k_1} + \frac{1}{k_2} \right)$$

$$\rightarrow C = \frac{2k_1 k_2}{k_1 + k_2} \frac{\epsilon_0 A}{d}$$

$$= 1.73 \times 10^{-11} \text{ F}$$



#51 A parallel-plate capacitor has a capacitance  $C = 100 \text{ pF}$ , a plate area of  $A = 100 \text{ cm}^2$  and a mica dielectric ( $K = 5.4$ ) completely filling the space between the plates. At  $50 \text{ V}$  potential difference, calculate (a) the electric field  $E$  in the mica, (b) the magnitude of the free charge  $Q$  on the plates and (c) the magnitude of the induced surface charge on the mica.

Solution

$$\text{Use } C = \frac{K \epsilon_0 A}{d} \rightarrow d = \frac{K \epsilon_0 A}{C}$$

$$\begin{aligned} \text{(a) } E &= \frac{V}{d} = \frac{VC}{K \epsilon_0 A} \\ &= 1.0 \times 10^4 \text{ V/m} \end{aligned}$$

$$(b) \quad Q = CV \\ = 5.0 \times 10^{-9} \text{ C}$$

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is  $q/2\epsilon_0 A$ , the field between the plates is

$$E = \frac{q_f}{2\epsilon_0 A} + \frac{q_f}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A}$$

where  $q_f$  is the free charge on the plates  
 $q_i$  is the induced charge on the <sup>dielectric</sup> surface

$$\begin{aligned} \rightarrow q_i &= q_f - \epsilon_0 A E \\ &= Q - \epsilon_0 A E \\ &= (5.0 \times 10^{-9} \text{ C}) - (8.85 \times 10^{-12} \text{ F/m}) \\ &\quad \times (100 \times 10^{-4} \text{ m}^2) (1.0 \times 10^4 \text{ V/m}) \\ &= \underline{\underline{4.1 \text{ nC}}} \end{aligned}$$