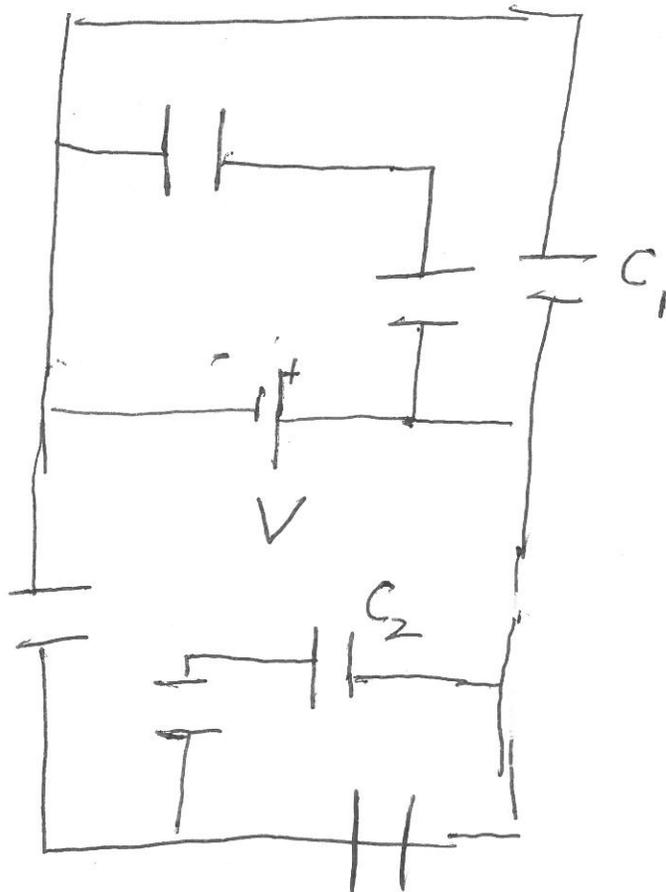


Assignment # 7

#56



In the Fig., the battery potential difference $V = 10.0 \text{ V}$ and each of the seven capacitors has capacitance $C = 10.0 \mu\text{F}$, what is the charge on (a) capacitor 1 and (b) capacitor 2?

Solution

(a) The potential across C_1 is 10 V. Therefore, the charge on it is

$$q_1 = C_1 V = (10.0 \mu\text{F})(10.0 \text{V}) \\ = 100 \mu\text{C}.$$

(b) Reducing the right portion of the circuit produces an equivalent capacitance equal to $6.0 \mu\text{F}$ with 10 V across it. Therefore, a charge of $60 \mu\text{C}$ is on it and consequently also on the bottom right capacitor. The bottom right capacitor has as a result a potential across it equal to

$$V = \frac{q}{C} = \frac{60 \mu\text{C}}{10 \mu\text{F}} = 6.0 \text{V}$$

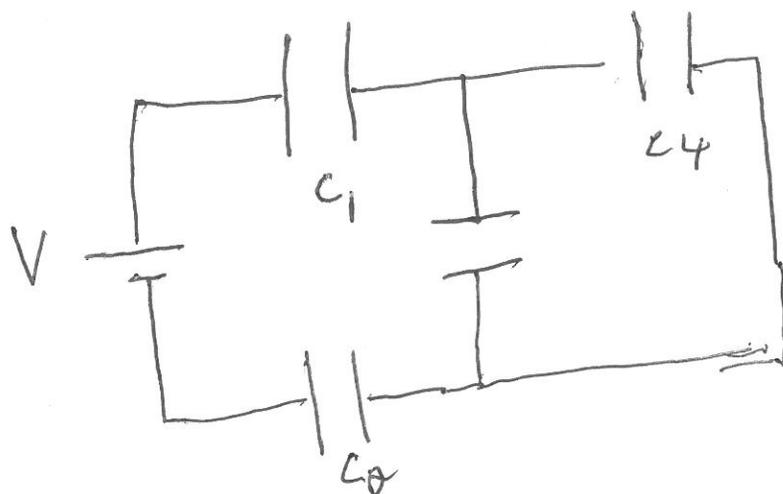
which leaves $(10.0\text{V} - 6.0\text{V}) = 4.0\text{V}$

across the group of capacitors in the upper right portion of the circuit. By inspection, the 4.0V must be equally divided by C_2 and the capacitance directly below it (in series with it). Therefore, with 2.0V across C_2 we have

$$q_2 = C_2 V_2 = (10.0\mu\text{F})(2.0\text{V}) \\ = 20.0\mu\text{C}$$

— || —

#57



Here, $V = 9.0\text{V}$, $C_1 = C_2 = 30\ \mu\text{F}$
and $C_3 = C_4 = 15\ \mu\text{F}$. What
is the charge on capacitor 4?

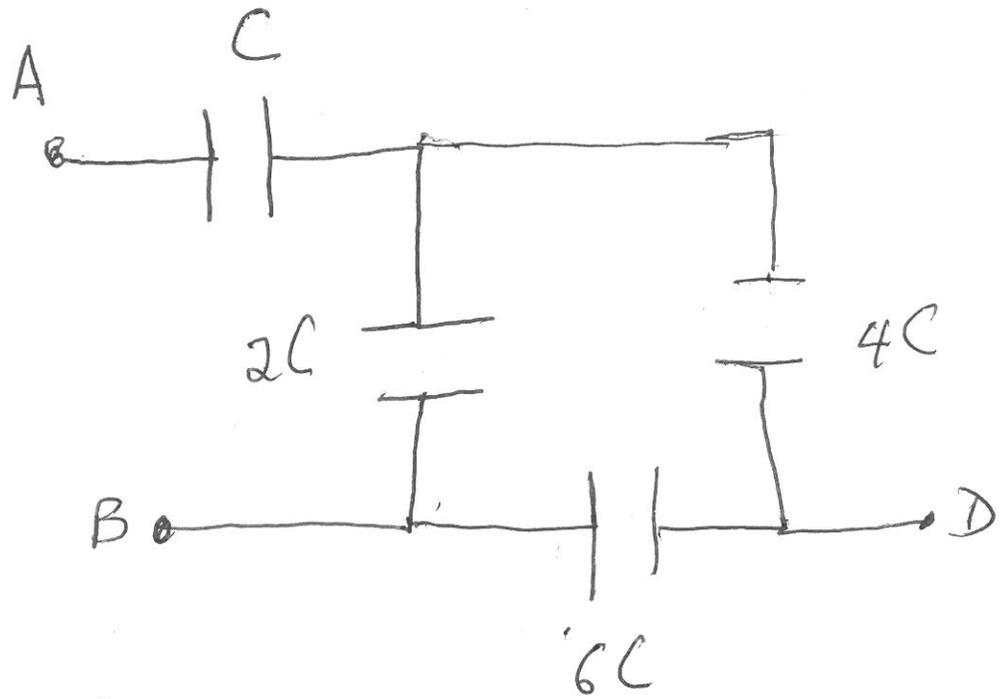
Solution

Since C_3 and C_4 are in parallel,
we replace them with an equivalent capacitance

$C_{34} = C_3 + C_4 = 30\ \mu\text{F}$. Now, C_1 , C_2
and C_{34} are in series and all have a
value of $30\ \mu\text{F}$. Thus, each has $\frac{1}{3}$
potential of the battery across it. Hence,
 3.0V across C_4 .

$$\begin{aligned} \rightarrow \text{charge on } C_4 \text{ is } q_4 &= (15\ \mu\text{F})(3.0\text{V}) \\ &= 45\ \mu\text{C} \end{aligned}$$

#58



(a) If $C = 50 \mu\text{F}$, what is the equivalent capacitance between A and B?

(b) Repeat for points A and D.

Solution

(a) The $4C$ and $6C$ capacitances are in series \rightarrow equivalent to $2.4C$. This is then in parallel with $2C$ which produces an equivalence of $4.4C$. The $4.4C$ is in series with $C \Rightarrow C_{\text{total}} = 0.82C = 41 \mu\text{F}$

(b) In (a) 'D' is not connected to anything.

Now, B is 'not' connected. So, $6C$

and $2C$ are now in series giving
an equivalent capacitance $1.5C$.

The $1.5C$ is in parallel with the $4C$

→ equivalent capacitance $5.5C$.

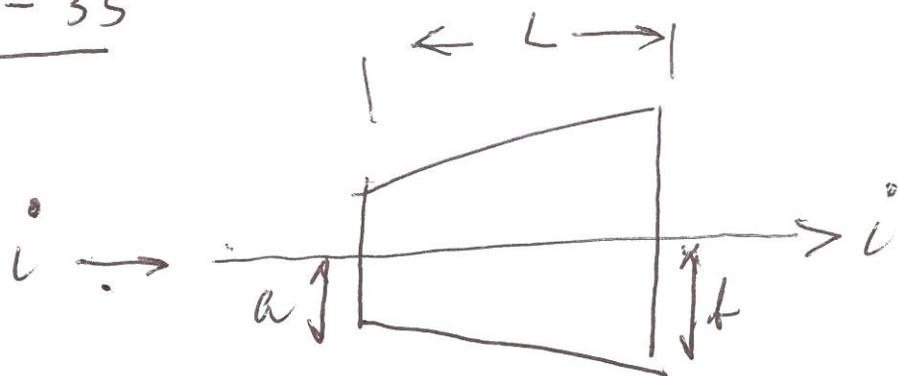
This $5.5C$ is in series with C

$$\rightarrow C_{\text{total}} = 0.85C$$

$$= 42 \mu\text{F}$$

— || —

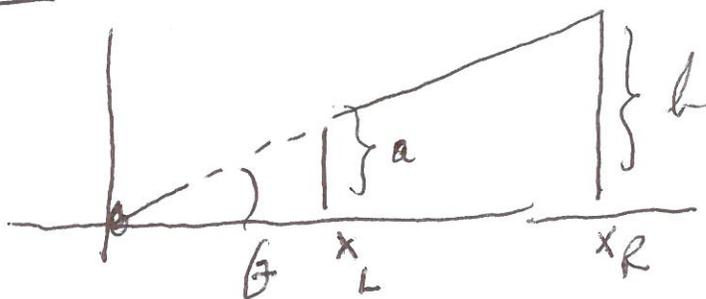
26-35



Current is flowing through a truncated right circular cone of resistivity $\rho = 731 \Omega \cdot \text{m}$, left radius $a = 2.0 \text{ mm}$, right radius $b = 2.3 \text{ mm}$ and length $L = 1.94 \text{ cm}$.

Assume that the current density is uniform any cross-section, taken perpendicular to the length. What is the resistance of the cone.

Solution



$$\tan \theta = \frac{b-a}{L}$$

$$x_L = \frac{a}{\tan \theta}, \quad x_R = \frac{b}{\tan \theta}$$

$$dR = \frac{\rho dx}{\pi r^2}; \quad r = x \tan \theta$$

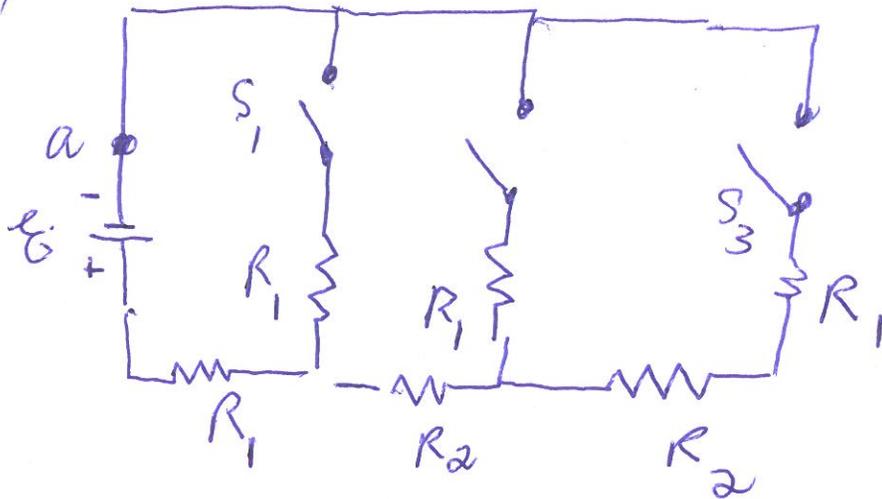
$$= \frac{\rho dx}{\pi x^2} \frac{1}{\tan^2 \theta}$$

$$\rightarrow R = \int dR = \frac{\rho}{\tan \theta} \frac{1}{\pi} \int_{x_L}^{x_R} \frac{dx}{x^2} = \frac{\rho}{\tan^2 \theta} \frac{1}{\pi} \left(\frac{1}{x_L} - \frac{1}{x_R} \right)$$

$$= \frac{1}{\pi} \frac{\rho}{\tan^2 \theta} \left(\frac{\tan \theta}{a} - \frac{\tan \theta}{b} \right)$$

$$= \frac{\rho}{\tan \theta} \frac{1}{\pi} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{\rho L}{\pi ab}$$

27-71



Here, $R_1 = 20\ \Omega$, $R_2 = 10\ \Omega$ and $\mathcal{E} = 120\text{V}$
 What is the current at 'a' if

- we close only switch S_1 ,
- only switches S_1 and S_2
- all three switches?

Solution

(a) If S_1 is closed and S_2 and S_3 are open then the current $i = \mathcal{E} / 2R_1 = \frac{120\text{V}}{40\ \Omega} = 3.0\text{A}$

(b) If S_1 and S_2 are closed while S_3 is open, then $R_{\text{eq}} = R_1 + \frac{R_1(R_1 + R_2)}{2R_1 + R_2} = 32\ \Omega$

and the current

$$i = \frac{\mathcal{E}}{R_{eq}} = \frac{120 \text{ V}}{32 \Omega} = 3.75 \text{ A}$$

(c) If all switches are closed, then

$$R_{eq} = R_1 + \frac{R_1 \tilde{R}}{R_1 + \tilde{R}}$$

where

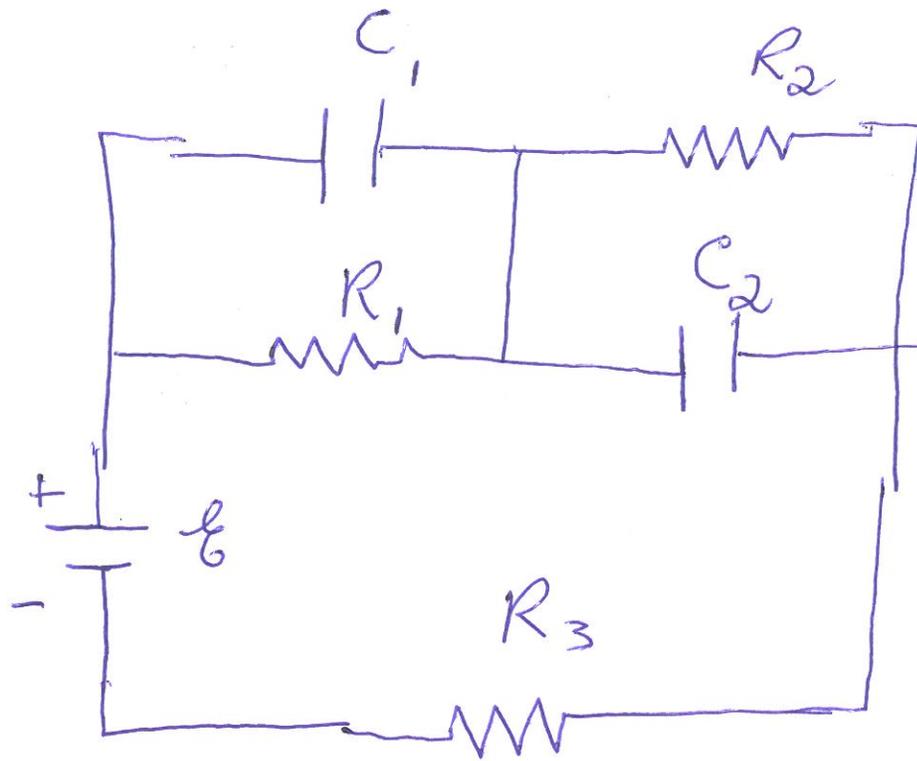
$$\begin{aligned} \tilde{R} &= R_2 + \frac{R_1 (R_1 + R_2)}{2R_1 + R_2} \\ &= 22 \Omega \end{aligned}$$

$$\begin{aligned} \rightarrow R_{eq} &= 20 \Omega + \frac{(20 \Omega)(22 \Omega)}{20 \Omega + 22 \Omega} \\ &= 30.5 \Omega \end{aligned}$$

$$\therefore i = \frac{\mathcal{E}}{R_{eq}} = \frac{120 \text{ V}}{30.5 \Omega} = 3.94 \text{ A}$$

————— || —————

27-80



$$R_1 = 5.0 \Omega, R_2 = 10.0 \Omega, R_3 = 15.0 \Omega, \\ C_1 = 5.0 \mu\text{F}, C_2 = 10.0 \mu\text{F}; \mathcal{E} = 20.0 \text{ V}.$$

Assuming the circuit is in the steady state, what is the ideal energy stored in the two capacitors?

Solution In the steady state, there is no current going through to the capacitors. In this case, the resistors all have the same current. Using Kirchoff's loop rule

$$20.0 \text{ V} = i(5.0 \Omega) + i(10.0 \Omega) + i(15.0 \Omega)$$

$$\rightarrow i = \frac{2}{3} \text{ A}$$

Therefore, the voltage across R_1 is $(5.0 \Omega)(\frac{2}{3} \text{ A}) = \frac{10}{3} \text{ V}$. This is the voltage across C_1 .

Therefore, the storage energy of C_1 is

$$U_1 = \frac{1}{2} C_1 V^2 = \frac{1}{2} (5.0 \times 10^{-6} \text{ F}) \left(\frac{10}{3} \text{ V}\right)^2 = 2.78 \times 10^{-5} \text{ J}$$

Similarly, the voltage across R_2 is

$$(10.0 \Omega) \left(\frac{2}{3} \text{ A}\right) = \frac{20}{3} \text{ V}$$

and is the voltage V_2 across C_2 .

$$\rightarrow U_2 = \frac{1}{2} C_2 V_2^2 = 2.22 \times 10^{-5} \text{ J}$$

→ So, total capacitance energy is

$$U = U_1 + U_2 =$$