HUNTER COLLEGE OF CUNY
Department of Physics
Physics 425

Intermediate Quantum Physics
Fall 2012
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November 26, 2012
Due: December 3, 2012

ASSIGNMENT # 5

1. In three dimensions, a particle of mass m^* is subject to the potential

$$V(r) = \begin{cases} -V_0 & r < a \\ 0 & r > a \end{cases} , \tag{1}$$

where

$$V_0 = \frac{8\hbar^2}{m^* a^2}$$

Recall that

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial}\left(r^2\frac{\partial}{\partial}\right) - \frac{\hat{L}^2}{\hbar^2 r^2}$$

(a) What is the largest possible squared-magnitude of angular momentum in a bound state of this system?

(b) For each possible outcome of measuring the squared-magnitude of angular momentum, give the range of possible bound state energies.

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2. A particle is subjected to a central potential $V(\vec{r}) = V(|\vec{r}|)$. We are told that it is in a superposition of eigenstates of eigenvalues $-2\hbar$, 0, $+\hbar$ with amplitudes $1/\sqrt{2}$, $1/\sqrt{6}$, $1/\sqrt{3}$, respectively.

(a) What is $< \hat{L}_z >$?

(b) What can one say about the outcome of a planned measurement of the component of angular momentum?

(c) What can one say about the outcome of a planned measurement of the squared-magnitude of angular momentum?

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3. Prove that the eigenvalues $l(l+1)\hbar^2$ of $\hat{L}^2$ could be obtained from the standard probability reasoning using the fact that the momentum projection to an arbitrary axis ξ could accept only the values $m = -l;\ -l+1;\ \ldots\ ;\ l-1;\ l$, having equal probability, and all the axes are equivalent.

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4. Find the form of the operators $\hat{L}L^2$ and $\hat{L}_z$ as well as their eigenvalues in the momentum representation. Prove that in the states with certain l and m, values the average momentum is zero.

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5. Demonstrate that the quantum averages of the $\vec{L}$, $\vec{p}$ and $\vec{r}$ operators in the state described as $\Psi(\vec{r}) = \phi(r)\mathsf{e}^{i\vec{p}_0\cdot\vec{r}/\hbar}$ satisfy the classical relation $\vec{L} = [\vec{r}\times\vec{p}]$. Here, $\vec{p}_0$ is an arbitrary constant vector and $\phi(r)$ is a function of r satisfying the following property: $\lim_{r\to\infty}\phi(r) \longrightarrow 0$

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6. Let Ψ_m be an eigenfunction of $\hat{L}_z$ with the eigenvalue m. Prove that the functions $\Psi_m^{\pm}$, which appear as $\Psi_m^{\pm} = \hat{L}^{\pm}\Psi_m$ are also eigenfunctions of $\hat{L}_z$ with the corresponding eigenvalues $m \pm 1$. Here, $\hat{L}^{\pm} = \hat{L}_x \pm i\hat{L}_y$.

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7. For the state $\Psi_{l,m}$ with angular momentum $l\hbar$ and its z-projection $m\hbar$, find the average values $\hat{L}_x^2$ and $\hat{L}_y^2$.