Superfluidity of excitons and polaritons in novel two-dimensional nanomaterials

Oleg L. Berman

Physics Department
New York City College of Technology of City University of New York (CUNY), Brooklyn, NY, USA

The Graduate School and University Center
City University of New York (CUNY)
New York, NY, USA
OUTLINE

• INTRODUCTION

• SUPERFLUIDITY OF DILUTE TWO-COMPONENT 2D DIPOLAR A and B EXCITONS IN TMDC DOUBLE LAYER

• DIRECTIONAL SUPERFLUIDITY OF DILUTE 2D DIPOLAR EXCITONS IN PHOSPHORENE DOUBLE LAYER

• SPIN HALL EFFECT FOR POLARITONS IN A TMDC MONOLAYER EMBEDDED IN A MICROCAVITY

• CONCLUSIONS
Nobel prize in Physics in 2001

- **Eric Cornell** and **Carl Wieman**, NIST and University of Colorado, Boulder.
- **Wolfgang Ketterle**, MIT

Bose-Einstein condensation

1925 → 1995

“This discovery must be viewed as one of the most beautiful physics experiments of the 20th century” - Lev Pitaevskii

The discovery of Bose-Einstein condensation (BEC) in 1995 in dilute, ultracold trapped atomic gases is one of the most exciting developments in recent physics research. In 2002, there are over 40 labs around the world which can routinely produce these atomic condensates.
Bose-Einstein condensation (ideal Bose gas)

Bose, 1924 $\rightarrow$ photons (mass $m = 0$) Einstein, 1925 $\rightarrow$ mass $m > 0$

Chemical potential $\mu = 0$

**BEC:** Chemical potential $\mu = 0$ at $T \leq T_c$

Chemical potential $\mu < 0$ at $T > T_c$

Distribution function $f(E)$

At very low temperatures ($T < T_c$)

At normal temperatures ($T > T_c$)
The Brief History of Bose-Einstein Condensation


1938: Pyotr L. Kapitsa (Nobel Prize 1978) discovered the superfluidity of $^4$He... the first experimental fingerprint of Bose-Einstein condensation in a dense system.

5. June 1995: the advent of BEC in trapped ultracold dilute atomic gases...

- $^{87}$Rb 5. June 1995  JILA (E. Cornell et al.)
- $^7$Li  July 1995  Rice Univ. (R. Hulet et al.)
- $^{23}$Na  Sept 1995  MIT (W. Ketterle et al.)
Making light atoms inside a solid

Excite electron-hole pair across a semiconductor band gap

Bound by the screened Coulomb interaction to make an exciton

\[ Ry^* = \frac{m^*/m}{\epsilon^2} \times \text{Rydberg} \]

\[ a_0^* = \frac{\epsilon}{m^*/m} \times a_{\text{Bohr}} \]

Excitons are the solid state analogue of positronium in GaAs
Binding energy \( \sim 5 \text{ meV} \)
Bohr Radius \( \sim 7 \text{ nm} \)

Polariton Effective Mass \( m^* \sim 10^{-4} m_e \)
\[ T_{\text{BEC}} \sim \frac{1}{m^*} \]

Combined coherent excitation is called a polariton
Quantum Well Excitons

Weakly bound electron-hole pair
EXCITON
Rydberg – few meV
Bohr Radius – few nm

Excitation spectrum
particle-hole continuum
QW exciton

in-plane center of mass momentum
Excitons + Cavity Photons

- Mirror
- QW
- Mirror

Energy vs. Momentum

QW exciton

Photon

Electron (e-)

Hole (h+)

C

V
Semiconductor microcavity structure

- Air
- Upper DBR
  - \( \lambda/4 \)-layer, refraction index \( n_1 \)
- Cavity spacer
  - One or more quantum wells
- Lower DBR
  - \( \lambda \)-layer, refraction index \( n_c \)
- Substrate
Polaritons: Matter-Light Composite Bosons

[C. Weisbuch et al., PRL 69 3314 (1992)]

\[ |\text{pol}\rangle = c_1 |\text{exc}\rangle + c_2 |\text{ph}\rangle \]

Effective Mass \( m^* \sim 10^{-4} m_e \)

\[ T_{\text{BEC}} \sim 1/m^* \]
Microcavity polaritons

Experiments:
Kasprzak et al 2006
CdTe microcavities

R. B. Balili, V. Hartwell, D. W. Snoke,
L. Pfeiffer and K. West,

GaAs microcavities
Direct excitons in quantum wells

Electron from conduction band + hole from valence band = exciton

Indirect excitons in GaAs/AlGaAs Coupled Quantum Wells (CQW)


Theory:

Coupled quantum wells

GaAs / AlGaAs

binding energy 4 meV
lifetime ~ 10 µs!
for D = 240 Å

spatial separation
gives long exciton
lifetime- 10 µs

but E-field gives
tunneling current
through the structure

aligned dipoles
give overall repulsion

Berman and Lozovik, JETP 84, 1027 (1997):

overall repulsive for D > 1.1a_B

2D indirect excitons
The spatially separated **electrons and holes** in a double layer.
ATOMIC WORLD versus EXCITATIONS WORLD

PHASES

- Nonideal gas
- Effects of Bose – Einstein statistics
- Bose – Einstein condensation atoms in trap
- Superfluid helium
  - Strong interaction: only 9% in Bose condensate
- Crystal phase

\[ T_c \approx 1 \text{nK} \quad \text{and} \quad T_c \approx 4 \text{K} \]

- Nonideal gas of excitons
- Manifestation of nonideality:
  - Blue shift of exciton line
- Manifestation of Bose induced processes was observed
- BEC of indirect excitons in CQW is claimed
- Exciton crystal phase is claimed
Graphene

Graphene was obtained and studied experimentally for the first time in 2004 by K. S. Novoselov, S. V. Morozov, A. K. Geim, et al. from the University of Manchester (UK).

A. K. Geim and K. S. Novoselov → Nobel Prize in Physics in 2010 for discovering graphene

BEC and superfluidity of dipolar excitons in a graphene double layer:


2D atomic honeycomb crystal lattice of carbon (graphite)

In CQWs:

1. Fluctuations of the width of QWs

2. Gap between conduction and valence bands = 1.43 eV (GaAsAlGaAs)

In graphene:

1. No fluctuations of the width of graphene layers (purely 2D)

2. No gap between conduction and valence bands.

Gap can be induced either by external electric or magnetic field or by impurities.

Perfect Graphene crystal and resultant Band Structure.

The effective mass of electrons and the energy gap in graphene equals 0

In semiconductor CQWs and graphene double layer: BEC and superfluidity of one-component dipolar excitons
Transition metal dichalcogenide (TMDC)

Review:


The structure of a TMDC monolayer.

BEC of dipolar excitons in a TMDC double ilayer:

BEC and superfluidity of two-component system of A and B dipolar excitons in a TMDC double layer:

Spatially separated electrons and holes in a TMDC double layer

M is a transition metal
X is a chalcogenide

1. Gap between valence and conduction bands: 1.6 -- 1.8 eV
2. No fluctuations of the width of TMDC layers.

h-BN monayers

$N_L$ number of monolayers

$D = N_L D_{hBN}$

thickness of one $h$-BN monolayer:

$D_{hBN} = 0.333$ nm
A and B 2D dipolar excitons in a TMDC double layer

The low-energy effective two-band single-electron Hamiltonian in the form of a spinor with a gapped spectrum for TMDCs in the $k \cdot p$ approximation:

$$
\hat{H}_s = a t (\tau k_x \hat{\sigma}_x + k_y \hat{\sigma}_y) + \frac{\Delta}{2} \hat{\sigma}_z - \lambda \tau \frac{\hat{\sigma}_z - 1}{2},
$$

$\hat{\sigma}$ denotes the Pauli matrices, $a$ is the lattice constant, $t$ is the effective hopping integral, $\Delta$ is the energy gap, $\tau = \pm 1$ is the valley index, $2\lambda$ is the spin splitting at the valence band top caused by the spin-orbit coupling (SOC), and $\hat{\sigma}_z$ is the Pauli matrix for spin $\hat{\sigma}$.

**Strong spin-orbit coupling (SOC) !!!**

$\Delta = 1.6 -- 1.8 \text{ eV}$

$2\lambda = 0.1 -- 0.5 \text{ eV}$


Significant spin-orbit splitting in the valence band leads to the formation of TMDC layers A and B in TMDC layers. Type A excitons are formed by spin-up electrons from conduction and spin-down holes from valence bands. Type B excitons are formed by spin-down electrons from conduction and spin-up holes from valence bands.


<table>
<thead>
<tr>
<th>Exciton type</th>
<th>MoS$_2$</th>
<th>MoSe$_2$</th>
<th>MoTe$_2$</th>
<th>WS$_2$</th>
<th>WSe$_2$</th>
<th>WTe$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.499</td>
<td>0.555</td>
<td>0.790</td>
<td>0.319</td>
<td>0.345</td>
<td>0.277</td>
</tr>
<tr>
<td>B</td>
<td>0.545</td>
<td>0.625</td>
<td>0.976</td>
<td>0.403</td>
<td>0.457</td>
<td>0.501</td>
</tr>
</tbody>
</table>
Two-body problem in Keldysh and Coulomb potentials

\[ M_A = m_{e\uparrow} + m_{h\downarrow} \]
\[ M_B = m_{e\downarrow} + m_{h\uparrow} \]

Electron and hole effective masses are taken from


**Keldysh potential (screening effects):**

\[
V_{eh}(r) = -\frac{\pi k e^2}{(\varepsilon_1 + \varepsilon_2) \rho_0} \left[ H_0 \left( \frac{r_{eh}}{\rho_0} \right) - Y_0 \left( \frac{r_{eh}}{\rho_0} \right) \right],
\]

\[
\varepsilon_1 = \varepsilon_2 = \varepsilon_d.
\]

\[
r_{eh} = \sqrt{r^2 + D^2}
\]

\[
\varepsilon_d = 4.89, \quad \text{screening length} \rho_0 = 2\pi \zeta / [(\varepsilon_1 + \varepsilon_2) / 2], \quad \text{where} \quad \zeta \quad \text{is the 2D polarizability.}
\]

**Coulomb potential (large \( D >\rho_0 \)):**

\[
V(r) = -\frac{k e^2}{\varepsilon_d r^2 + D^2}
\]

\( h\text{-BN monolayers} \quad \zeta = 4.1 \text{ Å} \)


**Analytical approach (2D harmonic oscillator approximation):**

Keldysh potential:

\[
V_0 = \frac{\pi k e^2}{2 \varepsilon_d \rho_0} \left[ H_0 \left( \frac{D}{\rho_0} \right) - Y_0 \left( \frac{D}{\rho_0} \right) \right],
\]

\[
\gamma = -\frac{\pi k e^2}{4 \varepsilon_d \rho_0^2 D} \left[ H_{-1} \left( \frac{D}{\rho_0} \right) - Y_{-1} \left( \frac{D}{\rho_0} \right) \right].
\]

**Coulomb potential:**

\[
V_0 = \frac{k e^2}{\varepsilon_d D^3}, \quad \gamma = \frac{k e^2}{2 \varepsilon_d D^3}
\]
A single dipolar exciton


Energy spectrum:

\[ E_{NL} \equiv E_{e(h)} = -V_0 + (2N + 1 + |L|) \hbar \left( \frac{2\gamma}{\mu} \right)^{1/2} \]

\( N = \min(\tilde{n}, \tilde{n}') \), \( L = \tilde{n} - \tilde{n}' \), \( \tilde{n}, \tilde{n}' = 0, 1, 2, 3, \ldots \) are the quantum numbers.

The reduced mass \( \mu = m_e m_h / (m_e + m_h) \)

Ground state energy:

\[ E_{00} = -V_0 + \hbar \left( \frac{2\gamma}{\mu} \right)^{1/2} \]

The harmonic oscillator approximation holds for

\[ D \gg D_0 \]

<table>
<thead>
<tr>
<th>Exciton</th>
<th>MoS(_2)</th>
<th>MoSe(_2)</th>
<th>WS(_2)</th>
<th>WSe(_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Mass/(m_0)</td>
<td>1.1</td>
<td>1.33</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>(D_0), Å</td>
<td>0.29</td>
<td>0.23</td>
<td>0.32</td>
</tr>
<tr>
<td>B</td>
<td>Mass/(m_0)</td>
<td>0.98</td>
<td>1.15</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>(D_0), Å</td>
<td>0.31</td>
<td>0.25</td>
<td>0.37</td>
</tr>
</tbody>
</table>

\( D_0 \) for A excitons is smaller than for B excitons

The energy spectrum of the center-of-mass of the A (or B) dipolar exciton \( \varepsilon_0^{A(B)}(\mathbf{P}) \) is given by

\[ \varepsilon_0^{A(B)}(\mathbf{P}) = \frac{P^2}{2M_{A(B)}} \]
## Binding energy of a dipolar exciton


<table>
<thead>
<tr>
<th>$N_L$</th>
<th>h-BN</th>
<th>Binding energy of A exciton, eV</th>
<th>Binding energy of B exciton, eV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MoS$_2$</td>
<td>MoSe$_2$</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>0.049</td>
<td>0.054</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>0.045</td>
<td>0.049</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>0.042</td>
<td>0.044</td>
</tr>
</tbody>
</table>

### A excitons:

<table>
<thead>
<tr>
<th>$N_L$</th>
<th>h-BN</th>
<th>Electron layer</th>
<th>Hole layer</th>
<th>Binding energy, eV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>MoS$_2$</td>
<td>MoSe$_2$</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>$B$, eV</td>
<td></td>
<td>0.050</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>$B$, eV</td>
<td></td>
<td>0.046</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>$B$, eV</td>
<td></td>
<td>0.042</td>
</tr>
</tbody>
</table>

### B excitons:

<table>
<thead>
<tr>
<th>$N_L$</th>
<th>h-BN</th>
<th>Electron layer</th>
<th>Hole layer</th>
<th>Binding energy, eV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>MoS$_2$</td>
<td>MoSe$_2$</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>$B$, eV</td>
<td></td>
<td>0.046</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>$B$, eV</td>
<td></td>
<td>0.043</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>$B$, eV</td>
<td></td>
<td>0.040</td>
</tr>
</tbody>
</table>

Binding energy for A excitons is greater than for B excitons for the same monolayers. The highest binding energy is in a MoSe$_2$ double layer, the lowest is in a WS$_2$ double layer.
Hamiltonian of two-component weakly interacting Bose gas of A and B dipolar excitons in a TMDC bilayer


Weakly interacting Bose gas of the dipolar excitons at low densities: \( na^2 \ll 1 \ (n \ll D^{-2}) \)

The Hamiltonian \( \hat{H} \) of the 2D A and B interacting dipolar excitons is given by

\[
\hat{H} = \hat{H}_A + \hat{H}_B + \hat{H}_I,
\]

where \( \hat{H}_{A(B)} \) are the Hamiltonians of A (B) excitons given by

\[
\hat{H}_{A(B)} = \sum_k E_{A(B)}(k)a_{kA(B)}^\dagger a_{kA(B)} + \frac{g_{AA(BB)}}{2S} \times \sum_{klm} a_{kA(B)}^\dagger a_{lA(B)}^\dagger a_A(B)m a_{A(B)k+l-m},
\]

and \( \hat{H}_I \) is the Hamiltonian of the interaction between A and B excitons given by

\[
\hat{H}_I = \frac{g_{AB}}{S} \sum_{klm} a_{kA}^\dagger a_{lB}^\dagger a_B m a_{Ak+l-m},
\]

where \( a_{kA(B)}^\dagger \) and \( a_{kA(B)} \) are Bose creation and annihilation operators for A (B) dipolar excitons with the wave vector \( k \), 
\( S \) is the area of the system,

\[
g_{AA} = g_{BB} = g_{AB} \equiv g
\]

We assume:

exciton-exciton dipole-dipole repulsion exists only at distances between excitons greater than distance from the exciton to the classical turning point.

The distance between two excitons cannot be less than this distance.
The collective excitations and sound velocity in a two-component dilute gas of A and B excitons

The Bogoliubov approximation for a two-component weakly interacting Bose gas: diagonalize the many-particle Hamiltonian.

The product of four operators is replaced by the product of two operators.

Most of the particles belong to BEC.

Only the interactions between the condensate and overcondensate particles are taken into account, the interactions between overcondensate particles are neglected.

The condensate operators are replaced by numbers.

The resulting Hamiltonian is quadratic with respect to the creation and annihilation operators.


\[
\varepsilon_j(k) = \sqrt{\frac{\omega_A^2(k) + \omega_B^2(k) + (-1)^j \sqrt{[\omega_A^2(k) - \omega_B^2(k)]^2 + 4g^2n^2\varepsilon_{(0)A}(k)\varepsilon_{(0)B}(k)}}{2}}.
\]

\[
\omega_A(k) = \sqrt{\varepsilon_{(0)A}^2(k) + gn\varepsilon_{(0)A}(k)}, \quad \omega_B(k) = \sqrt{\varepsilon_{(0)B}^2(k) + gn\varepsilon_{(0)B}(k)}.
\]

\[
n_A = n_B = n/2: \quad j = 1, 2
\]

\[
\mu_{AB} = \frac{M_A M_B}{M_A + M_B}.
\]

\[
c_j = \sqrt{\frac{gn}{2} \left( \frac{1}{2M_A} + \frac{1}{2M_B} + (-1)^j \sqrt{\left( \frac{1}{2M_A} - \frac{1}{2M_B} \right)^2 + \frac{1}{M_A M_B}} \right)}.
\]

\[
c = \sqrt{\frac{gn}{2} \left( \frac{1}{M_A} + \frac{1}{M_B} \right)}
\]

\(c\) is a sound velocity

\(c\) for another branch is zero
Superfluidity of two-component A and B dipolar excitons in a TMDC bilayer


The mean total current of 2D excitons in the coordinate system, moving with a velocity $u$

$$J = \int \frac{d^2p}{(2\pi\hbar)^2} p\{f[\varepsilon_1(p) - pu] + f[\varepsilon_2(p) - pu]\},$$

where $f[\varepsilon_1(p)] = \{\exp[\varepsilon_1(p)/(k_B T)] - 1\}^{-1}$ and $f[\varepsilon_2(p)] = \{\exp[\varepsilon_2(p)/(k_B T)] - 1\}^{-1}$ are the Bose-Einstein distribution

$$J = \rho_n u.$$

$\rho_n$ is the density of the normal component

$\rho_s$ is the density of the superfluid component

$$\rho_n(T_c) = \rho = M_A n_A + M_B n_B$$

$$T_c = \left[\frac{2\pi\hbar^2\rho}{3\xi(3)k_B^3\left(\frac{1}{c_1} + \frac{1}{c_2}\right)}\right]^{1/3}$$

$n_A = n_B = n/2$  \hspace{1cm} $T_c = \left[\frac{2\pi\hbar^2\rho c^4}{3\xi(3)k_B^3}\right]^{1/3}$

$T_c$ is a mean field critical temperature

For one-component system in CQWs or gapped graphene:

$$Q_A = \frac{8}{M_A} \text{ or } Q_B = \frac{8}{M_B}$$

$n/s \ s=4$ is the spin degeneracy factor

For two-component system in TMDC:

$$Q = \frac{M_A + M_B}{(\mu_{AB})^2}$$

For a one-component dilute exciton gas $Q^{1/3} n/s$ is always less than $Q^{1/3} n$ for a two-component Bose gas of A and B dipolar excitons. Therefore, $T_c$ is always higher for a two-component dilute dipolar exciton gas in TMDC than for a one-component dilute dipolar exciton gas in CQWs and gapped graphene.
**Q factor and critical temperature $T_c$**


**Keldysh potential:**

<table>
<thead>
<tr>
<th></th>
<th>MoS$_2$</th>
<th>MoSe$_2$</th>
<th>WS$_2$</th>
<th>WSe$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$, [1/$m_0$]</td>
<td>7.74</td>
<td>6.52</td>
<td>11.47</td>
<td>10.67</td>
</tr>
<tr>
<td>$\mu_{AB}$, [$m_0$]</td>
<td>0.52</td>
<td>0.62</td>
<td>0.36</td>
<td>0.39</td>
</tr>
<tr>
<td>$M_A + M_B$, [$m_0$]</td>
<td>2.08</td>
<td>2.48</td>
<td>1.46</td>
<td>1.49</td>
</tr>
</tbody>
</table>

**Critical Temperature $T_c$ (in K):**

\[ N_L = 10 \] of h-BN monolayers.

<table>
<thead>
<tr>
<th>Hole layer</th>
<th>MoS$_2$</th>
<th>MoSe$_2$</th>
<th>WS$_2$</th>
<th>WSe$_2$</th>
<th>MoS$_2$</th>
<th>MoSe$_2$</th>
<th>WS$_2$</th>
<th>WSe$_2$</th>
<th>MoS$_2$</th>
<th>MoSe$_2$</th>
<th>WS$_2$</th>
<th>WSe$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$, [1/$m_0$]</td>
<td>7.31</td>
<td>9.25</td>
<td>9.05</td>
<td>6.86</td>
<td>8.03</td>
<td>7.88</td>
<td>9.17</td>
<td>8.57</td>
<td>11.2</td>
<td>8.80</td>
<td>8.25</td>
<td>10.9</td>
</tr>
<tr>
<td>$\mu_{AB}$, [$m_0$]</td>
<td>0.55</td>
<td>0.44</td>
<td>0.45</td>
<td>0.59</td>
<td>0.50</td>
<td>0.52</td>
<td>0.44</td>
<td>0.47</td>
<td>0.37</td>
<td>0.46</td>
<td>0.49</td>
<td>0.38</td>
</tr>
<tr>
<td>$M_A + M_B$, [$m_0$]</td>
<td>2.21</td>
<td>1.77</td>
<td>1.82</td>
<td>2.35</td>
<td>2.04</td>
<td>2.09</td>
<td>1.77</td>
<td>1.90</td>
<td>1.51</td>
<td>1.85</td>
<td>1.98</td>
<td>1.54</td>
</tr>
</tbody>
</table>

The largest $Q$ and $T_c$ are in a WS$_2$ double layer, the smallest $Q$ and $T_c$ are in a MoSe$_2$ double layer.
Critical Temperature


Keldysh potential:

\[ N_L = 10 \text{ of } h\text{-BN monolayers.} \]

The largest \( T_c \) is in a WS\(_2\) double layer, the smallest \( T_c \) is in a MoSe\(_2\) double layer.
Critical Temperature for Keldysh and Coulomb potentials


\[ N_L = 10 \text{ of } h\text{-BN monolayers.} \]

\[ T_c \text{ for } \text{MoSe}_2 \text{ Keldysh potential} \]
\[ T_c \text{ for } \text{WS}_2 \text{ Keldysh potential} \]
\[ T_c \text{ for } \text{MoSe}_2 \text{ Coulomb potential} \]
\[ T_c \text{ for } \text{WS}_2 \text{ Coulomb potential} \]

\[ T_c \text{ for the Keldysh potential is smaller than for the Coulomb potential due to the screening effects.} \]
Critical Temperature for a WS$_2$ double layer


Keldysh potential:
Conclusions


- We propose a physical realization to observe high-temperature superconducting electron-hole currents in two parallel TMDC layers, caused by the superfluidity of quasi-two-dimensional dipolar A and B excitons in a TMDC double layer.

- The binding energies for A and B dipolar excitons in various TMDC double layers, taking into account screening effects by employing the approximated Keldysh potential, were obtained.

- The spectrum of collective excitations, obtained for a TMDC bilayer, is characterized by two branches due to the fact that the exciton system under consideration is a two-component weakly interacting Bose gas of A and B excitons in a TMDC double layer.

- The mean field critical temperature for the phase transition is analyzed for various TMDC materials.

- The mean field phase transition temperature for two-component dipolar excitons in a TMDC double layer is about one order of magnitude higher than for any one-component exciton system of semiconductor CQWs or gapped graphene.
2D electrons and holes in a phosphorene monolayer

Phosphorene is an atom-thick layer of black phosphorus

- natural band gap $\Delta \approx 2.2$ eV
- high hole mobility
- prominent anisotropic electron and hole effective masses, carrier mobility
- large excitonic binding energy $\sim 0.9$ eV


The single electron and hole energy spectrum for a phosphorene monolayer:

$$\varepsilon_l^{(0)}(\mathbf{p}) = \frac{p_x^2}{2m_x^l} + \frac{p_y^2}{2m_y^l}, \quad l = e, h,$$

where $m_x^l$ and $m_y^l$ are the electron/hole effective masses along the x and y directions, respectively. We assume that the OX and OY axes correspond to the armchair and zigzag directions in a phosphorene monolayer, respectively.

Electron and hole effective masses are taken from

A single dipolar exciton in a phosphorene double layer


A dipolar exciton consisting of a spatially separated electron and hole in a phosphorene double layer.

Hamiltonian

\[ \hat{H}_0 = -\frac{\hbar^2}{2m_e} \frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m_e} \frac{\partial^2}{\partial y_1^2} - \frac{\hbar^2}{2m_h} \frac{\partial^2}{\partial x_2^2} - \frac{\hbar^2}{2m_h} \frac{\partial^2}{\partial y_2^2} + V(\sqrt{r^2 + D^2}), \]

wave function \[ \Psi(\mathbf{R}, \mathbf{r}) = e^{i\mathbf{P} \cdot \mathbf{R}/\hbar} \varphi(\mathbf{r}) \]

the center of mass of an electron-hole pair \( \mathbf{R} = (X, Y) \)

\[ X = (m_e x_1 + m_h x_2)/(m_e + m_h), \quad Y = (m_e y_1 + m_h y_2)/(m_e + m_h), \]

the relative motion of an electron-hole pair

\[ \mathbf{r} = (x, y), \quad x = x_1 - x_2, \quad y = y_1 - y_2, \quad r^2 = x^2 + y^2 \]

Schrödinger equation:

\[ \left[ -\frac{\hbar^2}{2\mu_x} \frac{\partial^2}{\partial x^2} - \frac{\hbar^2}{2\mu_y} \frac{\partial^2}{\partial y^2} + V(\sqrt{r^2 + D^2}) \right] \varphi(x, y) = \mathcal{E} \varphi(x, y) \]

the reduced masses

\[ \mu_x = \frac{m_e m_h}{m_e + m_h} \quad \text{and} \quad \mu_y = \frac{m_e m_h}{m_e + m_h} \]
Two-body problem in Keldysh and Coulomb potentials

Insulator between two phosphorene monolayers:  \( h\text{-}BN \) monolayers

\[
D = N_L D_{hBN}
\]

thickness of one \( h\text{-}BN \) monolayer

\[
D_{hBN} = 0.333 \text{ nm}
\]

**Keldysh potential (screening effects):**

\[
V(r_{eh}) = -\frac{\pi ke^2}{(\varepsilon_1 + \varepsilon_2) \rho_0} \left[ H_0 \left( \frac{r_{eh}}{\rho_0} \right) - Y_0 \left( \frac{r_{eh}}{\rho_0} \right) \right].
\]

\[
\varepsilon_1 = \varepsilon_2 = \varepsilon_d.
\]

\[
r_{eh} = \sqrt{r^2 + D^2}
\]

\[
\varepsilon_d = 4.89 \quad \text{screening length} \quad \rho_0 = 2\pi \zeta / \left[ (\varepsilon_1 + \varepsilon_2) / 2 \right], \text{ where } \zeta \text{ is the 2D polarizability.}
\]

\( h\text{-}BN \) monolayers \( \zeta = 4.1 \text{ Å} \quad \text{Polarizability is taken from} \quad \text{A. S. Rodin, A. Carvalho, and A. H. Castro Neto, Phys. Rev. B 90, 075429 (2014).}

\( H_0(x) \) and \( Y_0(x) \) are Struve and Bessel functions of the second kind of order \( \nu = 0 \)

**Analytical approach (2D harmonic oscillator approximation):**

Keldysh potential:

\[
V_0 = \frac{\pi ke^2}{2\varepsilon_d \rho_0} \left[ H_0 \left( \frac{D}{\rho_0} \right) - Y_0 \left( \frac{D}{\rho_0} \right) \right],
\]

\[
\gamma = -\frac{\pi ke^2}{4\varepsilon_d \rho_0^2 D} \left[ H_{-1} \left( \frac{D}{\rho_0} \right) - Y_{-1} \left( \frac{D}{\rho_0} \right) \right].
\]

Coulomb potential (large \( D \gg \rho_0 \)):

\[
V(r) = -\frac{ke^2}{\varepsilon_d \sqrt{r^2 + D^2}}
\]

Insulator between two phosphorene monolayers:  \( h\text{-}BN \) monolayers

\[ N_L \text{ number of monolayers} \]

Insulator between two phosphorene monolayers:  \( h\text{-}BN \) monolayers

\[ N_L \text{ number of monolayers} \]
Effective masses and binding energy of a dipolar exciton

Energy spectrum of the center of mass of a single dipolar exciton:

\[ \varepsilon_0(P) = \frac{P_x^2}{2M_x} + \frac{P_y^2}{2M_y} \]

\[ \varepsilon_0(P) = \varepsilon_0(P, \Theta) = \frac{P^2}{2M_0(\Theta)} \]

\( P_x = P \cos \Theta \quad P_y = P \sin \Theta \quad M_x = m_x^e + m_x^h \) and \( M_y = m_y^e + m_y^h \) are the effective exciton masses, \( M_0(\Theta) \) is the effective angle-dependent exciton mass.

\[ M_0(\Theta) = \left[ \frac{\cos^2 \Theta}{M_x} + \frac{\sin^2 \Theta}{M_y} \right]^{-1} \]

TABLE II. The critical temperatures under the assumption about the soundlike spectrum of collective excitations for different sets of masses from Refs. [35–38]. The phosphorene layers are separated by 7 layers of \( h \)-BN. \( \mu_0 \) and \( M_x M_y \) are expressed in units of free electron mass \( m_0 \) and \( m_0^2 \), respectively.

<table>
<thead>
<tr>
<th></th>
<th>[35]</th>
<th>[36]</th>
<th>[37]</th>
<th>[38]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_0 \times 10^{-2} m_0 )</td>
<td>3.99</td>
<td>4.11</td>
<td>4.84</td>
<td>4.79</td>
</tr>
<tr>
<td>Coulomb potential ( T_c ) (K)</td>
<td>182</td>
<td>192</td>
<td>174</td>
<td>172</td>
</tr>
<tr>
<td>Keldysh potential ( T_c ) (K)</td>
<td>115</td>
<td>121</td>
<td>109</td>
<td>107</td>
</tr>
<tr>
<td>( M_x M_y \times m_0^2 )</td>
<td>1.67</td>
<td>1.23</td>
<td>2.24</td>
<td>2.39</td>
</tr>
</tbody>
</table>

Binding energy:

For the number \( N_L = 7 \) of \( h \)-BN monolayers, placed between two phosphorene monolayers, the binding energies of dipolar excitons, calculated for the sets of the masses from Refs. [35–38] by using Eq. (13), are given by 28.2 meV, 29.6 meV, 37.6 meV, and 37.2 meV.
Hamiltonian of a weakly interacting Bose gas of dipolar excitons in a phosphorene double layer


Weakly interacting Bose gas of the dipolar excitons at low concentrations:

The model Hamiltonian $\hat{H}$ of the 2D interacting dipolar excitons is given by

$$\hat{H} = \sum_P \varepsilon_0(P, \Theta) a_P^\dagger a_P + \frac{g}{S} \sum_{P_1P_2P_3} a_{P_1}^\dagger a_{P_2}^\dagger a_{P_3} a_{P_1+P_2-P_3},$$

where $a_P^\dagger$ and $a_P$ are Bose creation and annihilation operators for dipolar excitons with momentum $P$, $S$ is a normalization area for the system, $\varepsilon_0(P, \Theta)$ is the angular-dependent energy spectrum of noninteracting dipolar excitons,

We assume:

- exciton-exciton dipole-dipole repulsion exists only at distances between excitons greater than distance from the exciton to the classical turning point.
- The distance between two excitons cannot be less than this distance.

The Bogoliubov approximation for a weakly interacting Bose gas: diagonalize the many-particle Hamiltonian.

The product of four operators is replaced by the product of two operators.

Most of the particles belong to BEC.

Only the interactions between the condensate and noncondensate particles are taken into account.

The condensate operators are replaced by numbers.

The resulting Hamiltonian is quadratic with respect to the creation and annihilation operators.
The collective excitations and sound velocity in a dilute gas of dipolar excitons in phosphorene double layer


Hamiltonian $\hat{H}_{\text{col}}$ of the collective excitations in the Bogoliubov approximation for the weakly interacting gas of dipolar excitons in phosphorene is given by

$$\hat{H}_{\text{col}} = \sum_{P \neq 0, \Theta} \varepsilon(P, \Theta) \alpha_P^\dagger \alpha_P,$$

where $\alpha_P^\dagger$ and $\alpha_P$ are the creation and annihilation Bose operators for the quasiparticles with the energy dispersion corresponding to the angular-dependent spectrum of the collective excitations $\varepsilon(P, \Theta)$, described by

$$\varepsilon(P, \Theta) = \{[\varepsilon_0(P, \Theta) + \mu]^2 - \mu^2\}^{1/2}.$$  

In the limit of small momenta $P$, when $\varepsilon_0(P, \Theta) \ll gn$, $c_s(\Theta)$ is the angular-dependent sound velocity.

Since at small momenta the energy spectrum of the quasiparticles for a weakly interacting gas of dipolar excitons is soundlike, this means that the system satisfies the Landau criterion for superfluidity

$$\varepsilon(P, \Theta) = c_s(\Theta) P$$

$$c_s(\Theta) = \sqrt{\frac{gn}{M_0(\Theta)}}$$

The critical exciton velocity for superfluidity is angular dependent.

$$v_c(\Theta) = c_s(\Theta)$$
Angular Dependent Sound Velocity


Keldysh potential.

Coulomb potential.

\[ n = 2 \times 10^{16} \text{ m}^{-2} \]

number \( N_L = 7 \) of \( h \)-BN monolayers, placed between two phosphorene monolayers.
Directional Superfluidity of Dipolar Excitons in a Phosphorene Double Layer


The mean total mass current of 2D excitons in the coordinate system, moving with a velocity \( \mathbf{u} \)

\[
\mathbf{J} = s \int \frac{d^2 P}{(2\pi \hbar)^2} \mathbf{P} f[\varepsilon(P, \Theta) - \mathbf{P} \mathbf{u}].
\]

\( f[\varepsilon(P, \Theta)] = \{\exp[\varepsilon(P, \Theta)/(k_B T)] - 1\}^{-1} \) is the Bose-Einstein distribution function for quasiparticles

The normal density \( \rho_n \) in the anisotropic system has tensor form \( J_i = \rho_n^{(ij)}(T) u_j \).

For an anisotropic BCS superconductor: W. M. Saslow, Phys. Rev. Lett. 31, 870 (1973). The tensor of the concentration of the normal component

\[
n_n^{(ij)}(T) = \frac{s}{k_B M_i T} \int_0^\infty d P \frac{P^3}{(2\pi \hbar)^2}
\]

\[
\times \int_0^{2\pi} d\Theta \frac{\exp[\varepsilon(P, \Theta)/(k_B T)]}{\{\exp[\varepsilon(P, \Theta)/(k_B T)] - 1\}^2} \cos^2 \Theta
\]

\[
n_n^{(xy)}(T) = 0, \quad n_n^{(yx)}(T) = 0
\]

\[
\varepsilon(P, \Theta) = c_s(\Theta) P
\]

\[
n_n^{(xx)}(T) = n_n^{(yy)}(T) = \frac{2\zeta(3)s(k_B T)^3 \sqrt{M_x M_y}}{\pi (\hbar g n)^2}
\]

For an anisotropic BCS superconductor: \( n_n^{(xx)}(T) = n_n^{(yy)}(T) \neq n_n^{(yy)}(T) \).
Angular Dependence of Mean-Field Critical Temperature for Superfluidity I


\[
\tilde{n}_n(\Theta, T) = \sqrt{\left[n_n^{(xx)}(T)\right]^2 \cos^2 \Theta + \left[n_n^{(yy)}(T)\right]^2 \sin^2 \Theta}
\]

Keldysh potential.

\[
\varepsilon(P, \Theta) = \left[\varepsilon_0(P, \Theta) + \mu\right]^2 - \mu^2\right]^{1/2}
\]

number \(N_L = 7\) of h-BN monolayers, placed between two phosphorene monolayers.

Angular Dependence of Mean-Field Critical Temperature for Superfluidity II


\[ \tilde{n}_n(\Theta, T) = \sqrt{[n_n^{(xx)}(T)]^2 \cos^2 \Theta + [n_n^{(yy)}(T)]^2 \sin^2 \Theta} \]

\[ \tilde{n}_n(\Theta, T_c(\Theta)) = n. \]

\[ \varepsilon(P, \Theta) = \left\{ [\varepsilon_0(P, \Theta) + \mu]^2 - \mu^2 \right\}^{1/2} \]

Calculated for Coulomb potential

- 7 layers of h-BN, \( n = 2 \times 10^{11} \) cm\(^{-2}\)
- 7 layers of h-BN, \( n = 5 \times 10^{11} \) cm\(^{-2}\)
- 7 layers of h-BN, \( n = 1 \times 10^{12} \) cm\(^{-2}\)
- 7 layers of h-BN, \( n = 3 \times 10^{12} \) cm\(^{-2}\)

Conclusions

• The influence of the anisotropy of the dispersion relation of dipolar excitons in a double layer of phosphorene on the excitonic BEC and directional superfluidity has been investigated.

• The analytical expressions for the single dipolar exciton energy spectrum and wave functions have been derived.

• The anisotropy of the energy band structure in a phosphorene causes the critical velocity of the superfluidity to depend on the direction of motion of dipolar excitons.

• The dependence of the concentrations of the normal and superfluid components and the mean-field critical temperatures for superfluidity on the direction of motion of dipolar excitons occurs beyond the soundlike approximation for the spectrum of collective excitations.

• The directional superfluidity of dipolar excitons in a phosphorene double layer is possible.
Trapping Cavity Polaritons

P. Littlewood, Science 316, 989 (2007)

Cavity polaritons in TMDC:

Experiments:


cavity photon:

\[ E = \hbar c \sqrt{k_z^2 + k_{||}^2} = \hbar c \sqrt{(\pi / L)^2 + k_{||}^2} \]

quantum well exciton:

\[ E = E_{\text{gap}} - \Delta_{\text{bind}} + \frac{\hbar^2 N^2}{2m_r(2L)^2} + \frac{\hbar^2 k_{||}^2}{2m} \]
Semiconductor microcavity structure

Air

Upper DBR

Cavity spacer

Lower DBR

Substrate

$\lambda/4$-layer, refraction index $n_1$

$\lambda/4$-layer, refraction index $n_2$

One or more quantum wells

$\lambda$-layer, refraction index $n_c$
Elementary model of polariton modes

\[ H_k = \begin{pmatrix} E_k^{(cav)} & \hbar \Omega_R \\ \hbar \Omega_R^* & E_k^{(exc)} \end{pmatrix} \]

\[ E_k^{(\pm)} = \frac{E_k^{(cav)} + E_k^{(exc)}}{2} \pm \frac{1}{2} \sqrt{\left(E_k^{(cav)} - E_k^{(exc)}\right)^2 + 4 \left| \hbar \Omega_R \right|^2} \]

Realistic model includes finite polariton lifetime, leaky modes in the mirrors, etc.

We predict the Spin Hall effect (SHE) for microcavity polaritons in TMDC

1. The polaritons cloud is formed due to the coupling of excitons created in a TMDC layer and microcavity photons.

2. Two coordinate-dependent, counterpropagating and overlapping laser beams in the plane of the TMDC layer interact with a cloud of polaritons.

3. The counterpropagating and overlapping laser beams, characterized by Rabi frequencies $\Omega_1$ and $\Omega_2$, produce the spin-dependent gauge magnetic and electric fields due to strong SOC for electron and holes in TMDC.

4. The gauge magnetic field deflects the exciton component of polaritons consisting from the excitons with different spin states of charge carriers, namely $A$ and $B$ excitons, towards opposite directions.

5. For the laser pumping frequencies, corresponding to the resonant excitations of one type of excitons ($A$ or $B$), the corresponding excitons together with coupled to them photons form polaritons, which deflect to opposite transverse directions.

6. Using circular polarized pumping, one can excite both $A$ and $B$ excitons in one valley simultaneously.

7. In contrast, the superfluid components of the $A$ and $B$ polariton flows propagate in opposite directions along the counterpropagating beams.


Hamiltonian of microcavity polaritons in gauge fields

The Hamiltonian of TMDC polaritons in the presence of counterpropagating and overlapping laser beams:

$$\hat{H} = \hat{H}_{\text{exc}} + \hat{H}_{\text{ph}} + \hat{H}_{\text{exc-ph}} + \hat{H}_{\text{exc-exc}}$$

The Hamiltonian of TMDC excitons:

$$\hat{H}_{\text{exc}} = \sum_{\mathbf{P}} \varepsilon_{\text{exc}}(\mathbf{P}) \hat{b}_{\mathbf{P}}^\dagger \hat{b}_{\mathbf{P}}$$

$$\varepsilon_{\text{exc}}(\mathbf{P}) = E_{\text{bg}} - E_{\text{b}} + \varepsilon_0(\mathbf{P})$$

$E_{\text{bg}}$ is the band gap energy

$$\varepsilon_0(\mathbf{P}) = \frac{(\mathbf{P} - A_\sigma)^2}{2M}$$

$E_b$ is the binding energy of an exciton

where $M$ is the mass of an exciton and $A_\sigma$ is the gauge vector potential acting on the exciton component of polaritons, associated with different spin states of the conduction band electron, forming an exciton, $\sigma = \uparrow$ and $\downarrow$.

After applying the unitary transformations of the Hamiltonian at $H_{\text{exc-exc}} = 0$ →

The Hamiltonian of lower polaritons:

$$\hat{H}_0 = \sum_{\mathbf{P}} \varepsilon_{\text{LP}}(\mathbf{P}) \hat{p}_{\mathbf{P}}^\dagger \hat{p}_{\mathbf{P}}.$$

$$\varepsilon_{\text{LP}}(\mathbf{P}) = \hbar \pi q L_C^{-1} - |\hbar \Omega_R| + \varepsilon(\mathbf{P}).$$

$$\alpha \equiv 1/2(M^{-1} + (c/\tilde{n})L_C/\hbar \pi q)P^2/|\hbar \Omega_R| \ll 1$$

$$\varepsilon(\mathbf{P}) = \frac{(P - A_\sigma^{(\text{eff})})^2}{2M_p} + V^{(\text{eff})}$$

Effective mass of microcavity photons:

$$m_{\text{ph}} = \hbar \pi q/((c/\tilde{n})L_C)$$

Hamiltonian of exciton-photon coupling:

$$\hat{H}_{\text{exc-ph}} = \hbar \Omega_R \sum_{\mathbf{P}} \hat{a}_{\mathbf{P}}^\dagger \hat{b}_{\mathbf{P}} + H.c.$$

$\Omega_R$ is the Rabi splitting constant

Effective vector and scalar potentials, respectively, acting on polaritons,

$$A_\sigma^{(\text{eff})} \text{ and } V^{(\text{eff})}$$

$$A_\sigma^{(\text{eff})} = \frac{m_{\text{ph}} A_\sigma}{M + m_{\text{ph}}}, \quad V^{(\text{eff})} = \frac{A_\sigma^2}{4(M + m_{\text{ph}})}$$

$$M_p = 2\mu, \mu = M m_{\text{ph}}/(M + m_{\text{ph}})$$
The dependence of the effective gauge magnetic $B^{(\text{eff})}$ and electric $E^{(\text{eff})}$ fields on the parameter $l$.

$$B^{(\text{eff})}_\sigma = \nabla R \times A^{(\text{eff})}_\sigma = -\eta \frac{\hbar m_{\text{ph}} (|k_1| + |k_2|)}{4l(M + m_{\text{ph}})} e_z, \quad E^{(\text{eff})} = -\nablaRV^{(\text{eff})} = -\frac{\hbar^2 (|k_1| + |k_2|)^2}{16l(M + m_{\text{ph}})} e_y.$$ 

$\eta_\uparrow = 1$ for an $A$ exciton and $\eta_\downarrow = -1$ for a $B$ exciton.

Calculations performed for $|k_1| + |k_2| = 3 \, \text{µm}^{-1}$

$l = a^2/8y_0, a = 10 \, \text{µm}$ is the beam width.

$M$ is an exciton mass; $m_{\text{ph}}$ is a microcavity photon mass

$y_0$ is the spatial shift of two laser beams
Conductivity tensor for non-interacting polaritons

**Drude model:**

\[
\frac{dP}{dt} = E^{(\text{eff})} + \mathbf{v} \times B^{(\text{eff})} - \frac{P}{\tau} = M_P \mathbf{v}.
\]

For a steady state, setting \(dP/dt = 0\),

\[
P = M_P \mathbf{v}.
\]

\(\mathbf{v}\) is the velocity and \(\tau\) is a scattering time of microcavity polaritons.

Linear polariton flow density is defined as \(\mathbf{j} = n \mathbf{v}\)

\[
E^{(\text{eff})} = \frac{M_P}{n\tau} \mathbf{j} - \frac{\mathbf{j} \times B^{(\text{eff})}}{n}
\]

The conductivity tensor \(\tilde{\sigma}_{\sigma}\) is defined as the inverse matrix to the resistivity matrix \(\varrho_{\sigma}\).

\[
\sigma_{\sigma xx} = \frac{\sigma_0}{1 + \omega_c^2 \tau^2}, \quad \sigma_{\sigma xy} = -\sigma_{\sigma yx} = -\frac{\eta_{\sigma} \sigma_0 \omega_c \tau}{1 + \omega_c^2 \tau^2},
\]

\(\sigma_0 = \tau n/M_P\) and \(\omega_c = B^{(\text{eff})}/M_P\) is the cyclotron frequency.

\[l = a^2/8y_0, \quad a = 10 \, \mu m\] is the beam width.

Calculations performed for \(|k_1| + |k_2| = 3 \, \mu m^{-1}\)
Conductivity tensor for superfluid polaritons

Weakly interacting Bose gas of polaritons in dilute regime below $T_c$ is superfluid

$n_n$ is the concentration of the normal component

$n_s$ is the concentration of the superfluid component

$n$ is the total concentration

\[ n_s(T) = n - n_n(T) \]
\[ n_n(T) = \frac{3\zeta(3)}{2\pi \hbar^2} \frac{k_B^3 T^3}{c_s^4 M_p} \]

The polaritons in the superfluid component do not collide, $\tau \to +\infty$.

**Drude model:**

\[ M_p \frac{dv_x}{dt} = -\eta_\sigma B^{(eff)} v_y, \quad M_p \frac{dv_y}{dt} = E^{(eff)} + \eta_\sigma B^{(eff)} v_x \]

steady state, which corresponds to $dv_x/dt = dv_y/dt = 0$

\[ v_y = 0 \]
\[ v_x = -\frac{E^{(eff)}}{\eta_\sigma B^{(eff)}} \]

the linear superfluid polariton flow density $\mathbf{j}^{(s)} = n_s \mathbf{v}$.

conductivity tensor $\tilde{\sigma}_\sigma^{(s)}(T)$ for the superfluid

$\sigma_{\sigma,xx}^{(s)} = \sigma_{\sigma,yy}^{(s)} = 0, \quad \sigma_{\sigma,xy}^{(s)}(T) = -\sigma_{\sigma,yx}^{(s)}(T) = -\frac{n_s(T)}{\eta_\sigma B^{(eff)}}$

For the conductivity tensor $\tilde{\sigma}_\sigma^{(n)}(T)$ for the normal component

$\sigma_0(T) = \tau n_n(T)/M_p$

The sound spectrum of collective excitations:

with the sound velocity $c_s$

\[ \epsilon(P) = c_s P \]

**Linear polariton flow density**

1. Our calculations show that the contribution to the Hall linear polariton flow density from the superfluid component essentially exceeds the one from the normal component.

2. The contribution to the Hall linear polariton flow density from the superfluid component does not depend on the distance \( l \) between the counterpropagating laser beams.

3. The contribution to the linear polariton flow density from the superfluid component in the direction of the effective gauge electric field (\( y \) direction) is zero.

At the temperature \( T = 300 \text{ K} \) for \( \Delta \) polaritons we have obtained the total Hall linear polariton flow density in the presence of superfluidity:

- \( j_x^{(tot)} = 8.51887 \times 10^{13} \text{ nm}^{-1}\text{s}^{-1} \) for MoS\(_2\);
- \( j_x^{(tot)} = 9.54342 \times 10^{13} \text{ nm}^{-1}\text{s}^{-1} \) for MoSe\(_2\);
- \( j_x^{(tot)} = 9.76334 \times 10^{13} \text{ nm}^{-1}\text{s}^{-1} \) for WS\(_2\);
- \( j_x^{(tot)} = 1.1415 \times 10^{14} \text{ nm}^{-1}\text{s}^{-1} \) for WSe\(_2\).

**Proposed experiment**


1. The proposed experiment for observation of the spin Hall effect for microcavity polaritons is related to the measurement of the angular distribution of the photons escaping the optical microcavity.

2. In the absence of the effective gauge magnetic and electric fields, the angular distribution of the photons escaping the microcavity is central-symmetric with respect to the perpendicular to the Bragg mirrors.

3. We obtain the average tangent of the angles \( \alpha \) of deflection for the polariton flow in the \((x, y)\) plane of the microcavity:

\[
\tan \alpha = \left| \frac{j_x}{j_y} \right| = \left| \frac{\sigma_{o xy}}{\sigma_{o yy}} \right|
\]

**Without superfluidity:**

\[
\tan \alpha = \omega_c \tau
\]
Proposed experiment for observation of the SHE for microcavity polaritons in the presence of superfluidity

For the normal component:
\[ \tan \alpha^{(n)} = \left| \frac{j_x^{(n)}}{j_y^{(n)}} \right| = \left| \frac{\sigma_{\sigma xy}^{(n)}}{\sigma_{\sigma yy}^{(n)}} \right| = \omega_c \tau \]

For the superfluid component:
\[ \tan \alpha^{(s)} = \left| \frac{j_x^{(s)}}{j_y^{(s)}} \right| = \left| \frac{\sigma_{\sigma xy}^{(s)}}{\sigma_{\sigma yy}^{(s)}} \right| \rightarrow +\infty \]
\[ \sigma_{\sigma yy}^{(s)} = 0 \]
\[ \overline{\alpha}^{(s)} = 90^0 \]

Calculations performed for \( |k_1| + |k_2| = 3 \mu m^{-1} \)
\[ l = \frac{a^2}{8y_0}, a = 10 \mu m \text{ is the beam width}, \]
\[ \tan \alpha^{(n)} \approx 10^{-5}, \text{ and } \overline{\alpha}^{(n)} \approx 10^{-3} \]


SHE for microcavity polaritons allows to separate the superfluid component from the normal components of the polariton flow!
Conclusions


• We have predicted the spin Hall effect for microcavity polaritons, formed by excitons in a TMDC and microcavity photons.

• We demonstrated that the polariton flow can be achieved by generation the effective gauge vector and scalar potentials, acting on polaritons.

• We have obtained the components of polariton conductivity tensor for both non-interacting polaritons without BEC and for weakly-interacting Bose gas of polaritons in the presence of BEC and superfluidity.

• We demonstrated that due to the SHE the polariton flows in the same valley are splitting: the normal components of the $A$ and $B$ polariton flows slightly deflect in opposite directions and propagate almost perpendicularly to the counterpropagating beams, while the superfluid components of the $A$ and $B$ polariton flows propagate in opposite directions along the counterpropagating beams.

• We predicted the method to separate the superfluid component from the normal component of the polariton flow due to the SHE.
I wish to thank my co-authors

Prof. Godfrey Gumbs
Hunter College, CUNY

Prof. Roman Ya. Kezerashvili
City Tech, CUNY

Prof. Yurii E. Lozovik
Institute of Spectroscopy