Geometry

1) Find the center and radius of the circle with equation \(x^2 + y^2 - 6x + 10y + 9 = 0\).

Solution: Start by putting the equation into standard form by completing the square.

\[
x^2 - 6x + y^2 + 10y = -9 \Rightarrow x^2 - 6x + 9 + y^2 + 10y + 25 = -9 + 9 + 25 \Rightarrow (x - 3)^2 + (y + 5)^2 = 25 = 5^2
\]

The center is (3, -5) and the radius is \(r = 5\).

2) Evaluate the piecewise function at the given values of the independent variable.

\[
f(x) = \begin{cases} 
\frac{x^2 - 9}{3}, & \text{if } x \neq 3 \\
6, & \text{if } x = 3 
\end{cases}
\]

Evaluate \(f(5), f(0)\) and \(f(3)\).

Solution: \(f(5) = \frac{(5)^2 - 9}{3} = \frac{25 - 9}{3} = \frac{16}{3}\)

\(f(0) = \frac{0^2 - 9}{3} = \frac{-9}{3} = -3\)

\(f(3) = 6\).

3) Let \(A(-7, 4)\) and \(B(5, -12)\) be points in the plane. Find the equation of the line through the points \(A\) and \(B\).

Solution: \(m = \frac{\Delta y}{\Delta x} = \frac{-12 - 4}{5 - (-7)} = -\frac{16}{12} = -\frac{4}{3}\)

Point slope form: \(y - 4 = -\frac{4}{3}(x + 7)\)

Slope intercept form: \(y = -\frac{4}{3}x - \frac{16}{3}\).
4) Find the domain of the piecewise function \( f(x) = \begin{cases} 
 0, & \text{if } x < -4 \\
 2, & \text{if } x \geq 0 \\
 -x, & \text{if } -4 \leq x < 0
\end{cases} \)

**Solution:** Domain = \((-\infty, -4) \cup [-4, 0) \cup [0, \infty) = (-\infty, \infty).\)

5) Sketch the region in the xy-plane defined by \(|x| < 4\) and \(|y| < 2\).

6) Find the domain of \( g(x) = \frac{\sqrt{x-2}}{x-5} \).

**Solution:** \( g \) is defined at \( x \) under the condition that \( x \neq 5 \) and \( x - 2 \geq 0 \Rightarrow x \neq 5 \) and \( x \geq 2 \).

Domain: \([(-\infty, 5) \cup (5, \infty)] \cap [2, \infty) = [2, 5) \cup (5, \infty).\)

7) Identifying even and odd functions:

a) \( f(x) = x^3 - 6x \)

**Solution:** \( f(-x) = (-x)^3 - 6(-x) = -x^3 + 6x = -(x^3 - 6x) = -f(x) \)

Therefore, \( f \) is an odd function.

b) \( g(x) = x^4 - 2x^2 \)

**Solution:** \( g(-x) = (-x)^4 - 2(-x)^2 = x^4 - 2x^2 = g(x) \)

Therefore, \( g \) is an even function.

c) \( h(x) = x^2 + 2x + 1 \)

**Solution:** \( h(-x) = (-x)^2 + 2(-x) + 1 = x^2 - 2x + 1 \)

\( H \) is neither even nor odd.
8) Find the equation of the line that passes through the point (2, -5) and a) has a slope of -3.

   Solution: Point-slope form: $y + 5 = -3(x - 2)$
   
   Slope-intercept form: $y = -3x + 1$

b) Is parallel to the y-axis.

   Solution: $x = 2$

c) Is parallel to the line $2x - 4y = 3$

   Solution: Slope of $2x - 4y = 3$ is $m = \frac{1}{2}$. Since we want our line to be parallel to the given line, we need the same slope.
   
   Point-slope form: $y + 5 = \frac{1}{2}(x - 2)$
   
   Slope-intercept form: $y = \frac{1}{2}x - 6$.

Circles

1) Find the equation of the circle with center at (1, -5) and radius of 3.

   Solution: $(x – 1)^2 + (x + 5)^2 = 9$

2) Given the equation of a circle, $(x – 3)^2 + (y + 4)^2 = 3$. Find its center and radius.

   Solution: Center: (1, -5), radius: $\sqrt{3}$

3) Give the center and radius of the circle defined by $x^2 + (y – 3)^2 = 9$.

   Solution: Center: (0, 3), radius: 3

4) Give the radius and center of the circle defined by $3x^2 + 3y^2 = 9$.

   Solution: Center: (0, 0), radius: $\sqrt{3}$. 
5) Find the equation of the circle with center at (-6, -7) and radius \( r = 6 \).

**Solution:** 
\[(x + 6)^2 + (y + 7)^2 = 36.\]

**Parallel/Perpendicular**

1) Find the equation of the line passing through (3, 0) and perpendicular to the x-axis.

**Solution:** The equation is \( x = 3 \).

2) Find the equation of the line passing through (-4, 5) and parallel to the x-axis.

**Solution:** The equation is \( y = 5 \).

3) Find the equation of the line containing (0, -2) and parallel to the x-axis.

**Solution:** The equation is \( y = -2 \).

4) Find the equation of the line passing through (-4, 5) and parallel to the y-axis.

**Solution:** The equation is \( x = -4 \).

5) Find the equation of the line that is parallel to the graph of \( y = 3x - 2 \) and has the y-intercept (0, 3).

**Solution:** The equation is \( y = 3x + 3 \).

6) Find the equation of the line that is perpendicular to the graph of \( y = 5 \) and has the y-intercept (0, 2).

**Solution:** The equation is \( x = 0 \).
7) Find the equation of the line that is perpendicular to the graph of $x = -2$ and passes through the origin.

**Solution:** The equation is $y = 0$. 