COMBINING RATIONAL EXPRESSIONS

A rational expression is a quotient of polynomials such as $\frac{3x^2-4x+2}{x^3+2x^2-4}$. To combine rational expressions such as $\frac{2x+1}{x^2-4}-\frac{3x+2}{x^2-6x+8}$, we follow the same steps as we did for fractions, namely:

- Find the least common denominator (LCD)
- Rewrite each fraction as an equivalent fraction with the LCD
- Combine the numerators

(See the brush-up materials for operations on fractions if you need a review.)

Remember that when working with rational expressions, we can never have a value for a variable that will make our denominator zero.

Example 1: $\frac{2}{x+3}+\frac{3}{x}$

Solution:
The LCD of the two denominators is $x(x+3)$.
Rewriting each fraction as an equivalent fraction with this LCD, we get:

$$\frac{2}{x+3}=\frac{2x}{(x)x+3} \text{ and } \frac{3}{x}=\frac{3(x+3)}{x(x+3)}=\frac{3x+9}{x(x+3)}$$

Combining the numerators, we get: $\frac{2x}{(x)x+3}+\frac{3x+9}{x(x+3)}=\frac{5x+9}{x(x+3)}$

When combining rational expressions, sometimes finding the LCD can be a challenging step. Look at the next example to see why.

Example 2: $\frac{2x+1}{x^2-4}-\frac{3x+2}{x^2-6x+8}$

Solution:
To find the LCD of $\frac{2x+1}{x^2-4}-\frac{3x+2}{x^2-6x+8}$, we first factor the denominators to obtain $\frac{2x+1}{(x+2)(x-2)}-\frac{3x+2}{(x-4)(x-2)}$.

The LCD must include the factors $(x+2),(x-4),\text{and}(x-2)$. Each fraction can be written as an equivalent fraction with the denominator $(x+2)(x-4)(x-2)$. Altogether,

$$\frac{2x+1}{(x+2)(x-2)}-\frac{3x+2}{(x-4)(x-2)}=\frac{(2x+1)}{(x+2)(x-2)}\cdot\frac{(x-4)}{(x-4)}-\frac{(3x+2)}{(x-4)(x-2)}\cdot\frac{(x+2)}{(x+2)}=$$

$$\frac{(2x+1)(x-4)-(3x+2)(x+2)}{(x+2)(x-2)(x-4)}=\frac{2x^2-7x-4-(3x^2+8x+4)}{(x+2)(x-2)(x-4)}=$$

$$\frac{2x^2-7x-4-3x^2-8x-4}{(x+2)(x-2)(x-4)}=\frac{-x^2-15x-8}{(x+2)(x-2)(x-4)}\ .$$

IMPORTANT:

- It is necessary to put a numerator that contain more than one term in parentheses when you are finding an equivalent fraction.
- Be sure that when you multiply out an expression like $(3x+2)(x+2)$ that you keep this result in parentheses so that you then subtract the entire quantity.
- Try to factor the numerator, to see if any cancellation is possible. In this case, the numerator does not factor.

Practice Exercises:

1. Find the LCD of the following groups of fractions:

a) $\frac{2}{5x}, \frac{3x}{4x^2}, \text{and} \frac{4}{3}$ b) $\frac{3}{x^2+x-6} \text{and} \frac{5x}{2x+6}$

2. Simplify:

a) $\frac{5}{3x}-\frac{2}{x^2}+\frac{3}{2x}$ b) $\frac{3}{x+5}-\frac{1}{x}$ c) $\frac{3}{y+6}+\frac{4}{y-3}$ d) $\frac{x}{x-4}+\frac{5}{x+5}-\frac{11x-8}{x^2+x-20}$

e) $5-\frac{x-2}{x+1}$ f) $\frac{4x+1}{x-8}-\frac{3x+2}{x+4}-\frac{49x+4}{x^2-4x-32}$

Answers:

1. a) $60x^2$ b) $2(x+3)(x-2)$

2. a) $\frac{19x-12}{6x^2}$ b) $\frac{2x-5}{x(x+5)}$ c) $\frac{7y+15}{(y+6)(y-3)}$
   d) $\frac{x+3}{x+5}$ e) $\frac{4x+7}{x+1}$ f) $\frac{x-2}{x+4}$