INTRODUCTION TO EXPONENTS AND LOGARITHMS

Exponents

When you raise a number to a "power", you are raising it to an *exponent.* So an exponent is the same thing as a power. For example, in the expression

$$2^5$$

the **5** is the exponent. It means, in this case, to multiply 2 by itself five times.

$$2 \, x \, 2 \, x \, 2 \, x \, 2 \, x \, 2 = 2^5$$

On your calculator, this can generally be accomplished by using either the Y^x key or possibly the ^ key.

Try it:

$$2 \, [Y^x] \, 5 = 32$$

$$2 \, [\, \wedge \,] \, 5 = 32$$

Logarithms

A logarithmic expression is very closely related to an exponential expression. When you evaluate a logarithmic expression, you are trying to *find an exponent*. The following example,

$$\log_{10} 100$$

is a way of referring to the number ***that would raise 10 to become 100.*** In fact,

$$\log_{10} 100 = 2$$

is a true statement precisely because $10^2 = 100$. So the logarithmic expression above, $\log_{10} 100$, can actually be thought of as asking, "What exponent must we put on 10 in order for it to become 100?"

The number in subscript is called a *base.* When there is no base, you assume that the base is 10. For instance, the example above might have been written *log 100 = 2.*

Sometimes, bases may be numbers other than 10. Here is another example:

$$\log_3 81 = 4$$

This is because $3^4 = 81$. In this example, the base is 3.

There is a special number called e. It is **approximately** 2.718. (Actually, we wouldn't be able to write it out fully since it has decimals that go on forever.) It has special properties that make it useful in various areas in mathematics. When it is used as a base, the resulting log is called a *natural log.* Instead of using the word log and writing a base, it would be written simply with **ln**. An equation using a natural log might look like

$$\ln e = 1$$

Remember that the base in the above example is unwritten; it can be thought of this way:

$$\log_e e = 1$$

So it can still be thought of in the same way as the earlier examples of logs. It can be interpreted as asking, "What exponent must we put on e in order for it to become e?" Of course, the answer to that question is, $e^1 = e$.

Try to find the answer to this example without looking:

$$\ln e^3 = ?$$

Ask yourself the question that was used above to clarify the meaning of logarithms: "What exponent must we put on e in order for it to become e^3?" The answer is built into the question itself. We must put an exponent of 3 on e for it to become e^3.

If our rounded value of e is squared, we would have $2.718^2 = 7.387524$. That is why we can say (approximately) that

$$\ln 7.387524 = 2$$

because *the exponent we must put on e (2.718) in order to make it become 7.387524* is **2**.

Here are some exercises:

1. $\log_4 16$
2. $\log_2 64$
3. $\log_7 49$
4. $\log_9 81$
5. $\log_3 9$
6. $\log_3 81$
7. $\log 1000$
8. $\log_{10} 1$

9. $\log_5 1/5$	13. $10^{\log 100}$
10. $\log_{10} 10^8$	14. $e^{\ln 100}$
11. $\ln e^7$	15. $\log_5 1/25$
12. $\log_4 4^3$	16. $\log_{10} .01$

Answers

1. 2	5. 2	9. -1	13. 100
2. 6	6. 4	10. 8	14. 100
3. 2	7. 3	11. 7	15. -2
4. 2	8. 0	12. 3	16. -2