**WORKING WITH EXPONENTS (including negative and rational exponents)**

Before working with exponents we must refresh our memory of the rules for exponents which are listed below:

| Rules for exponents: | Examples: |
|---|---|
| 1) $x^0 = 1,\ x \neq 0$ | 1) $(x+2y)^0 = 1$ |
| 2) $x^{-n} = \frac{1}{x^n}$ | 2) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ |
| 3) $x^n \cdot x^m = x^{n+m}$ | 3) $x^3 \cdot x^4 = x^7$ |
| 4) $\frac{x^n}{x^m} = x^{n-m}$ | 4) $\frac{x^5}{x^{-3}} = x^{5-(-3)} = x^8$ |
| 5) $(xy)^n = x^n y^n$ | 5) $(xy)^{-2} = x^{-2}y^{-2} = \frac{1}{x^2 y^2}$ |
| 6) $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ | 6) $\left(\frac{x}{y}\right)^{\frac{1}{2}} = \frac{x^{\frac{1}{2}}}{y^{\frac{1}{2}}}$ |
| 7) $(x^n)^m = x^{nm}$ | 7) $(x^3)^5 = x^{15}$ |

**Model Problems:**

(1) $(-2a^2b^3)^2$

$(-2a^2b^3)^2 = (-2)^2 a^{2\cdot 2} b^{3\cdot 2} = 4a^4b^6$

(Notice that the -2 must be put in parenthesis to get the correct sign of the final answer.

(2) $(3x^{-2}y^4)^{-3}$

$(3x^{-2}y^4)^{-3} = 3^{-3}x^6y^{-12} = \frac{x^6}{3^3 y^{12}} = \frac{x^6}{27y^{12}}$

Here, the final answer should have only positive exponents; also notice that to evaluate a number raised to a negative exponent, the exponent must first be made positive.

(3) $\frac{1}{x^{-2}}$

$\frac{1}{x^{-2}} = \frac{1}{\frac{1}{x^2}} = x^2$

To make the exponent positive you just have to change its position in the fraction (from numerator to denominator, or as in the example above, from the denominator to the numerator).

(4) $\left(\frac{2x^{-5}y^2}{x^{-7}y^5}\right)^{-3}$

$$\left(\frac{2x^{-5}y^{2}}{x^{-7}y^{5}}\right)^{-3}=\left(2x^{-5-(-7)}y^{2-5}\right)^{-3}=\left(2x^{2}y^{-3}\right)^{-3}=2^{-3}x^{-6}y^{9}=\frac{y^{9}}{2^{3}x^{6}}=\frac{y^{9}}{8x^{6}}$$

(5) $8^{\frac{4}{3}}$

$$8^{\frac{4}{3}}=(8^{\frac{1}{3}})^{4}=2^{4}=16$$

It's often easier to take the root first and then raise it to the power.

(6) $16^{-\frac{1}{4}}$

$$16^{-\frac{1}{4}}=\frac{1}{16^{\frac{1}{4}}}=\frac{1}{2}$$

Remember, first make the exponent positive before you evaluate a number raised to a negative exponent.

(7) $\dfrac{x^{\frac{1}{6}}x^{-\frac{2}{3}}}{x^{\frac{1}{2}}}$

$$\frac{x^{\frac{1}{6}}x^{-\frac{2}{3}}}{x^{\frac{1}{2}}}=\frac{x^{\frac{1}{6}}x^{-\frac{4}{6}}}{x^{\frac{3}{6}}}=\frac{x^{-\frac{3}{6}}}{x^{\frac{3}{6}}}=x^{-\frac{3}{6}-\frac{3}{6}}=x^{-\frac{6}{6}}=x^{-1}=\frac{1}{x}$$

Here, a common denominator for all the fractional exponents is found, the numerator is simplified, and then the subtraction rule is used. The final answer has a positive exponent.

**Practice Exercises:**

1. $(x^{2}y^{3})^{4}$
2. $\dfrac{x^{-5}}{x^{6}}$
3. $(-3x^{-3}y^{2})^{-2}$
4. $\dfrac{(2x^{-3}y^{2})^{-2}}{x^{2}y^{-3}}$
5. $\left(\dfrac{4x^{2}y^{-2}}{6x^{3}y^{2}}\right)^{-2}$
6. $27^{\frac{2}{3}}$
7. $81^{-\frac{3}{4}}$
8. $\dfrac{3^{\frac{1}{6}}3^{-\frac{1}{4}}}{3^{\frac{1}{2}}}$

**Answers:**

1. $x^{8}y^{12}$
2. $\dfrac{1}{x^{11}}$
3. $\dfrac{x^{6}}{9y^{4}}$
4. $\dfrac{x^{4}}{4y}$
5. $\dfrac{9x^{2}y^{8}}{4}$
6. $9$
7. $\dfrac{1}{27}$
8. $\dfrac{1}{3^{\frac{7}{12}}}$