WORKING WITH INTERVAL NOTATION, LINEAR INEQUALITIES AND ABSOLUTE VALUE INEQUALITIES

Interval Notation
Interval notation is used to represent subsets of the real numbers. There are finite and infinite intervals, and finite intervals sometimes include one or both endpoints.

Model Problems:

(1) Represent all numbers between -2 and 4: $(-2,4)$
The smaller number is always on the left.
In set notation, this interval would be represented as $\{x|-2<x<4\}$.
This is read as "the set of numbers x such that x is greater than -2 and less than 4".

(2) Represent all numbers between -10 and 3, including 3: $(-10,3]$
If a number is included, a square bracket is used.
In set notation, this interval is represented as $\{x|-10<x\le 3\}$.

(3) Represent all numbers greater than -4: $(-4,\infty)$
This is an example of an infinite interval.
A square bracket is never used for ∞ or $-\infty$.
In set notation this interval is represented as $\{x|\ x>-4\}$

The set of all real numbers is written $(-\infty,\infty)$.

Linear Inequalities and Absolute Value Inequalities
There are 4 types of inequalities:

$>$ greater than
$<$ less than
$\ge$ greater than or equal to
$\le$ less than or equal to

Model Problem:

Solve: $-4(x-2)+3\le -2(x-2)$

First simplify both sides:
$$-4x+8+3\le -2x+4$$
$$-4x+11\le -2x+4$$

Add $2x$ on both sides and also subtract 11 on both sides: $-2x\le -7$

Divide both sides by -2 remembering that when you multiply or divide an inequality by a negative number, you must switch the direction of the inequality: $x\ge \frac{7}{2}$

In interval notation, the answer is $\left[\frac{7}{2}, \infty\right)$

Absolute value linear inequalities

There are basically two types of absolute value linear inequalities: $>$ (or $\geq$) and $<$ (or $\leq$).

Model Problems:

(1) Solve: $|2x-4|<10$

Absolute value can be interpreted as distance from the origin 0. So in this case, we want $2x-4$ to have distance less than 10 from 0. That is, we want $2x-4$ to lie between -10 and 10:

$$-10<2x-4<10$$

This is an example of a compound inequality: $2x-4$ is both greater than -10 and less than 10.

Adding 4 to all three parts of the inequality: $-6<2x<14$

Dividing by 2: $-3<x<7$

In interval notation the answer is $(-3,7)$.

(2) Solve: $|3x-4|>2$

Here we want $3x-4$ to have distance greater than 2 from the origin 0. That is, $3x-4$ must be to the left of -2 or to the right of 2.

$$|3x-4|>2$$

	$3x-4<-2$	or	$3x-4>2$
Solving:	$3x<2$	or	$3x>6$
	$x<\frac{2}{3}$	or	$x>2$

The answer is a union of 2 intervals: $\left(-\infty,\frac{2}{3}\right)\cup(2,\infty)$.

Practice Exercises:

1. Put the following sets of numbers into interval notation.

   a) all numbers between 5 and 15, including 5

   b) all numbers greater than -2

   c) all numbers that are at least 3

   d) all numbers that are at most 22

2. Solve the following linear inequalities. Put your answers in interval notation.

a) $2-3(2x-4)>-3(x+5)-10$

b) $\frac{2x}{3}-\frac{x+5}{4}\leq -x+\frac{1}{6}$ (Hint: first multiply by the LCD to eliminate all denominators)

3. Solve the following absolute value inequalities. Put your answers in interval notation.

a) $|-2x-4|<5$

b) $|3x+4|\geq 6$

c) $|5x-4|-2\leq 6$ (Hint: first isolate the absolute value expression)

Answers:

1. a) $[5,15)$ b) $(-2,\infty)$ c) $[3,\infty)$ d) $(-\infty,22]$

2. a) $(-\infty,13)$ b) $(-\infty,1]$

3. a) $\left(-\frac{9}{2},\frac{1}{2}\right)$ b) $\left(-\infty,-\frac{10}{3}\right)\cup\left[\frac{2}{3},\infty\right)$ c) $\left[-\frac{4}{5},\frac{12}{5}\right]$.