

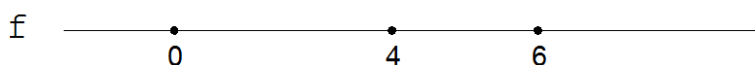
## WORKING WITH POLYNOMIAL AND RATIONAL INEQUALITIES

When you graph functions in a Calculus class, you are asked to find where the function is increasing and where it is decreasing. This is one place where you need to be able to solve polynomial and rational inequalities. Below, we do an example of each.

### Model Problems:

Example 1: Find the intervals on which  $x(x-4)(x-6) > 0$ .

To solve a polynomial inequality, first make sure that 0 is on one side of the inequality, and the other side appears in factored form. Determine where the polynomial expression is equal to 0, and then draw a number line which includes each of these numbers.



These numbers determine intervals,  $(-\infty, 0)$ ,  $(0, 4)$ ,  $(4, 6)$ , and  $(6, \infty)$ , where the polynomial expression  $x(x-4)(x-6)$  is either always positive or always negative. To determine the sign of the polynomial expression for each interval, we choose a number inside each interval and note the sign of the polynomial expression at that number. This will determine the sign of the polynomial expression for the entire interval:

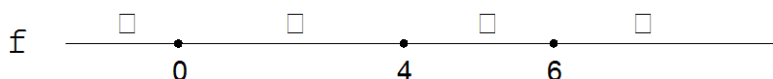
$$f(-1) = (-1)(-1-4)(-1-6) = (-1)(-5)(-7) = -35 < 0.$$

$$f(1) = 1(1-4)(1-6) = (-3)(-5) = 15 > 0$$

$$f(5) = 5(5-4)(5-7) = 5(1)(-2) = -10 < 0$$

$$f(7) = 7(7-4)(7-6) = 7(3)(1) = 21 > 0$$

So our number line looks like:



Therefore,  $x(x-4)(x-6) > 0$  on  $(0, 4) \cup (6, \infty)$ .

To solve a rational inequality, there is the additional step of including the number at which the denominator is equal to 0. This is usually indicated on the number line as an empty circle because it cannot be part of any solution set.

Example 2: Find the intervals on which  $\frac{x-5}{(x-1)^3}$  is positive and where it is negative.

As before, we draw a number line according to where the expression is equal to 0, and

additionally according to where the expression is undefined.



We have three intervals on which to determine the sign of our rational expression:  $(-\infty, 1)$ ,  $(1, 5)$ , and  $(5, \infty)$ . To determine the sign on each of these intervals, we choose a number inside each of the intervals, and evaluate the expression at the chosen number:

$$f(0) = \frac{0-5}{(0-1)^3} = \frac{-5}{-1} = 5 > 0 \quad f(2) = \frac{2-5}{(2-1)^3} = \frac{-3}{1} = -3 < 0, \quad f(6) = \frac{6-5}{(6-1)^3} = \frac{1}{125} > 0.$$

So our number line looks like:



Therefore,  $\frac{x-5}{(x-1)^3} > 0$  on  $(-\infty, 1) \cup (5, \infty)$  and  $\frac{x-5}{(x-1)^3} < 0$  on  $(1, 5)$ .

### Practice Exercises:

1. Solve the following inequalities.

(a)  $(x+5)^2(x-2) > 0$

(b)  $x^2 - 4x < -3$  (remember to first get 0 on one side of the inequality)

(c)  $x^3 - 6x^2 - 4x + 24 > 0$  (factor the left side by the method of grouping – this is discussed in the factoring section)

(d)  $\frac{(3x-2)(x-4)}{(x+4)^2} < 0$

### Answers:

1. (a)  $(2, \infty)$  (b)  $(1, 3)$  (c)  $(-2, 2) \cup (6, \infty)$  (d)  $(\frac{2}{3}, 4)$

