Workshop Exercises: Techniques of Integration I

1. Integrate by the method of integration by parts.

a) $\int x e^x \, dx$.

b) $\int x \sin x \, dx$.

c) $\int x^2 \sin x \, dx$.

d) $\int x^3 \ln x \, dx$.

e) $\int_0^1 \tan^{-1} x \, dx$.

f) $\int e^{2x} \cos x \, dx$.

g) $\int x^5 \sqrt{x^3 + 4} \, dx$.

2. Integrate the following trigonometric integrals.

a) $\int \cos x \sin^2 x \, dx$.

b) $\int \cos^3 x \sin^2 x \, dx$.

c) $\int \cos^3 x \sin^3 x \, dx$.

d) $\int \sin^2 x \cos^2 x \, dx$.

e) $\int \tan^3 x \sec^2 x \, dx$ (or $\int \cot^3 x \csc^2 x \, dx$).

f) $\int \tan^3 x \sec^4 x \, dx$ (or $\int \cot^3 x \csc^4 x \, dx$).

g) $\int \cot^3 x \csc^3 x \, dx$ (or $\int \tan^3 x \sec^3 x \, dx$).

h) $\int \cot^2 x \csc x \, dx$ (or $\int \tan^2 x \sec x \, dx$).

3. Integrate by the method of trigonometric substitution.

a) $\int \frac{x^3}{\sqrt{9-x^2}} \, dx$.

b) $\int \frac{1}{(2-x^2)^{\frac{3}{2}}} \, dx$.

c) $\int \frac{\sqrt{4-x^2}}{x} \, dx$.

d) $\int_0^4 \frac{x^3}{(x^2+9)^{\frac{3}{2}}} \, dx$.

e) $\int \frac{1}{\sqrt{1+4x^2}} \, dx$.

f) $\int_2^3 \frac{1}{x^2 \sqrt{x^2-1}} \, dx$.

g) $\int \frac{1}{\sqrt{x^2+4x+3}} \, dx$.

h) $\int \sqrt{5 - 4x - x^2} \, dx$.

Solutions:

1. a) $x e^x - e^x + C$

   b) $-x \cos x + \sin x + C$

   c) $-x^2 \cos x + 2 x \sin x + 2 \cos x + C$

   d) $\frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + C$

   e) $\frac{\pi}{4} - \frac{\ln 2}{2}$

   f) $\frac{e^{2x}}{5} (\sin x + 2 \cos x) + C$

   g) $\frac{2}{9} x^3 (x^3 + 4)^{\frac{3}{2}} - \frac{4}{45} (x^3 + 4)^{\frac{5}{2}} + C$

2. a) $\frac{1}{3} \sin^3 x + C$

   b) $\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$

   c) $\frac{1}{4} \sin^4 x - \frac{1}{6} \sin^6 x + C =$
   $\frac{1}{6} \cos^6 x - \frac{1}{4} \cos^4 x + C$

   d) $\frac{1}{8} x - \frac{1}{32} \sin 4 x + C$

   e) $\frac{1}{4} \tan^4 x + C$

   f) $\frac{1}{4} \tan^4 x + \frac{1}{6} \tan^6 x + C$

   g) $-\frac{1}{5} \csc^5 x + \frac{1}{3} \csc^3 x + C$

   h) $-\frac{1}{2} \csc x \cot x - \frac{1}{2} \ln \left| \csc x - \cot x \right| + C$

3. a) $-9 (9 - x^2)^{\frac{1}{2}} + \frac{1}{3} (9 - x^2)^{\frac{3}{2}} + C$

   b) $\frac{x}{2\sqrt{2-x^2}} + C$

   c) $2 \ln \left| \frac{2-\sqrt{4-x^2}}{x} \right| + \sqrt{4 - x^2} + C$

   d) $\frac{4}{5}$

   e) $\frac{1}{2} \ln \left| \sqrt{1 + 4 x^2} + 2 x \right| + C$

   f) $\frac{2\sqrt{2}}{3} - \frac{\sqrt{3}}{2}$

   g) $\ln \left| x + 2 + \sqrt{x^2 + 4 x + 3} \right| + C$

   h) $\frac{9}{2} \sin^{-1}\left(\frac{x+2}{3}\right) + \frac{x+2}{2} \sqrt{5 - 4 x - x^2} + C$