Workshop Exercises: Applications of the Integral

1. a) Find the area bounded by $y = x^2 - 2$ and $y = x + 4$.

   b) Find the area bounded by $y = x^{\frac{1}{3}}$ and $y = \frac{x}{4}$. (Note: this area consists of two regions).

   c) Find the area bounded by $x = 2y$ and $y^2 = x + 3$.

2. Find the volume of the solid generated by revolving the area bounded by the curves $y = 2x^2$ and $y = x^3$ about

   a) the x - axis, using the washer method.

   b) the y - axis, using the shell method.

   c) the y - axis, using the washer method.

   d) the line $x = -1$, using the shell method.

3. Find the volume of the solid generated by revolving the area bounded by the curves $x = 2y$ and $y^2 = x + 3$ about the line $y = 4$.

4. A force of 120 pounds is required to maintain a spring 6 feet beyond its natural length.

   a) Find the work done in stretching the spring 2 feet beyond its natural length.

   b) Find the work done in stretching the spring from 4 feet to 6 feet beyond its natural length.

5. A tank whose sides are isosceles triangles (vertex down) with height 4 feet, base 6 feet, and length 10 feet is full of water that weighs $62.5\frac{\text{lb}}{\text{ft}^3}$. Find the work done in pumping the water

   a) to the top of the tank.

   b) to an outlet 1 feet above the top of the tank.

   c) to an outlet 1 feet above the top of the tank if the tank is filled only half-way to the top.