Workshop Exercises: Arc Length and Surface Area

1. Find the length of the curve.

a) $y = \left(4 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}, \quad 1 \le x \le 27.$

b) $y = \frac{2}{3}\,(x^2 + 1)^{\frac{3}{2}}, \quad 1 \le x \le 2.$

c) $x = \frac{y^4}{16} + \frac{1}{2\,y^2}\,, \quad -3 \le y \le -2.$

d) $y = \ln(\sin x)\,, \quad \frac{\pi}{6} \le x \le \frac{\pi}{2}.$

2. Find the area of the surface obtained by rotating the curve about the x-axis.

a) $y = \frac{x^3}{3}, \quad 0 \le x \le 1.$

b) $y = \frac{x^3}{6} + \frac{1}{2\,x}\,, \quad 1 \le x \le 2.$

c) $x = \frac{y^4}{4} + \frac{1}{8\,y^2}\,, \quad 1 \le y \le 2.$

3. Find the area of the surface obtained by rotating the curve about the y-axis.

a) $y = \frac{x^2}{2} + 1\,, \quad 0 \le x \le 1.$

b) $y = \frac{1}{24}\,(x^2 + 16)^{\frac{3}{2}}, \quad 0 \le x \le 3.$

c) $x = \sqrt{25 - y^2}\,, \quad 0 \le y \le 3.$