Workshop Exercises: Differentiation and Integration Review

1. A force of 120 pounds is required to maintain a spring 6 feet beyond its natural length.

   a) Find the work done in stretching the spring 2 feet beyond its natural length.

   b) Find the work done in stretching the spring from 4 feet to 6 feet beyond its natural length.

2. A tank whose sides are isosceles triangles (vertex down) with height 4 feet, base 6 feet, and length 10 feet is full of water that weighs $62.5 \frac{lb}{ft^3}$. Find the work done in pumping the water

   a) to the top of the tank.

   b) to an outlet 1 feet above the top of the tank.

   c) to an outlet 1 feet above the top of the tank if the tank is filled only half-way to the top.

3. Differentiate the following functions.

   a) $f(x) = e^{2x}$.

   b) $f(x) = \ln(2x)$.

   c) $h(x) = 3^{1/x}$.

   d) $f(x) = (\cos(x)) \cdot 2^{-4x}$.

   e) $f(x) = \sin^{-1}(x^2)$.

   f) $g(x) = (\tan^{-1}(2x))^3$.

   g) $f(x) = \log_4 \sqrt{x^2 + 1}$.

4. Differentiate using logarithmic differentiation.

   a) $y = (3x+2)^3 (4x-5)^5$.   c) $y = x^{\ln(x)}$.

   b) $y = x^{\cos(x)}$.   d) $y = (\ln(x))^x$.

5. A bacterial population starts with 10,000 bacteria and grows at a rate proportional to its size.
   After 2 hours there are 40,000 bacteria.

   a) Find the number of bacteria after 5 hours.

   b) When will the population reach 1 million?

6. (Carbon Dating) All living things contain carbon-12, which is stable, and carbon-14, which is radioactive. While a plant or animal is alive, the ratio of these two isotopes of carbon remains unchanged since the carbon-14 is constantly renewed; after death, no more carbon-14 is absorbed. The half-life of carbon-14 is 5,730 years. If changed logs of an old fort showed only 70% of the carbon-14 expected in living matter, when did the fort burn down? Assume the fort burned soon after it was built of freshly sawed logs.

7. Evaluate the integral for each of the following.

   a) $\int \frac{(\ln x)^2}{x} \, dx$.   d) $\int e^x \, dx$

   b) $\int \frac{\sin 2x}{1 + \cos^2 x} \, dx$.   e) $\int x^3 \, e^{x^4} \, dx$

   c) $\int \frac{\cos x}{2 + \sin x} \, dx$.   f) $\int_0^1 e^{-x} \, dx$