Workshop Exercises: Techniques of Integration II

1. Integrate by the method of partial fractions.

a) $\int \frac{1}{x^2-5x+6}\,dx.$

b) $\int \frac{x-3}{x^2-3x+2}\,dx.$

c) $\int \frac{x^3+x^2}{x^2+5x+6}\,dx.$

d) $\int \frac{x^2+8x-4}{x^3-4x}\,dx.$

e) $\int \frac{2x-3}{(x-1)^2}\,dx.$

f) $\int \frac{1}{(x^2-1)(x-1)}\,dx.$

g) $\int \frac{x^2-x-4}{x(x^2+4)}\,dx.$

h) $\int \frac{x^3-20x+4}{(x^2+4)^2}\,dx.$

2. Integrate by using a rationalizing substitution.

a) $\int \frac{1}{1+\sqrt{x}}\,dx.$

b) $\int x\sqrt{1+x}\,dx.$

c) $\int \frac{x^{\frac{1}{2}}}{4\left(1+x^{\frac{3}{4}}\right)}\,dx.$

d) $\int \frac{e^{2x}}{\sqrt{1+e^x}}\,dx.$

3. Use (a) the Midpoint Rule, (b) the Trapezoidal Rule, and (c) Simpson's Rule with $n = 6$ to approximate $\int_0^3 \frac{1}{1+x^3}\,dx$. (Round your answers to four decimal places).

4. a) Approximate $\int_1^3 \ln x\,dx$ using the Trapezoidal Rule and Simpson's rule with $n = 4$. (Round your answers to four decimal places).

   b) Estimate the errors in the approximations of part (a). (Round your answers to three decimal places).

5. Evaluate the given improper integral or show that it diverges.

a) $\int_1^\infty \frac{x}{\sqrt{9+x^2}}\,dx.$

b) $\int_2^\infty \frac{1}{x(\ln x)^2}\,dx.$

c) $\int_{-\infty}^\infty \frac{x}{(x^2+4)^2}\,dx.$

d) $\int_{-\infty}^\infty \frac{1}{x^2+4x+5}\,dx.$

e) $\int_0^3 \frac{x}{\sqrt{9-x^2}}\,dx.$

f) $\int_2^4 \frac{1}{(3-x)^2}\,dx.$