Workshop Exercises: Polar Coordinates

1. The polar coordinates of a point are given.

   (i) Plot each point.
   (ii) Find two other pairs of polar coordinates for each point, one with $r > 0$ and one with $r < 0$.
   (iii) Find the Cartesian coordinates of the point.

   a) $\left(2, \frac{\pi}{3}\right)$. b) $\left(-1, \frac{2\pi}{3}\right)$. c) $\left(\sqrt{2}, -\frac{5\pi}{4}\right)$. d) $\left(-3, -\frac{\pi}{6}\right)$.

2. Find polar coordinates of the points with the given Cartesian coordinates.

   a) $(4, 4)$. b) $(-3, 0)$. c) $\left(\sqrt{3}, -1\right)$. d) $(0, 2)$.

3. Sketch the curve with the given equation and then find the indicated area.

   a) $r = 3 + 3\cos\theta$ (cardiod); find the area enclosed by the curve.

   b) $r = 4\cos 3\theta$ (three-leaved rose); find the area enclosed by one loop.

   c) $r^2 = 9\sin 2\theta$ (lemniscate); find the area enclosed by one loop.

   d) $r = 3 - 6\sin\theta$ (limacon); find the area enclosed by its inner loop.

4. Find the area of the region that lies inside the lemniscate $r^2 = 8\cos 2\theta$ and outside the circle $r = 2$.

5. Find the slope of the line tangent to the curve $r = 2\sin\theta$ at $\theta = \frac{\pi}{6}$.