

# Workshop Exercises: Sequences and Introduction to Infinite Series

1. Determine whether the sequence converges or diverges. If it converges, find the limit.

a)  $a_n = \frac{3n-1}{2n+1}$ .

f)  $a_n = \cos(n\pi)$ .

b)  $a_n = \frac{-2n^3 - 4n + 2}{n^2 + 3}$

g)  $a_n = \cos\left(\frac{\pi}{n}\right)$ .

c)  $a_n = \frac{3n^2}{2n^2(4n+5)}$ .

h)  $a_n = e^{-n} \cos(n\pi)$ .

d)  $a_n = \frac{n}{\sqrt{4n^2 - n + 2}}$ .

i)  $a_n = \frac{\ln n}{\sqrt{n}}$ .

e)  $a_n = \frac{(-1)^n (2n+1)}{5n-3}$

j)  $a_n = (2n)^{\frac{1}{2n}}$ .

2. Find a formula for a general term  $a_n$  of the sequence, and then determine whether it converges or diverges.

a)  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$

b)  $2, 1, \frac{2^3}{3^2}, \frac{2^4}{4^2}, \frac{2^5}{5^2}, \dots$

3. Determine whether the given series converges or diverges. If it converges, find its sum.

a)  $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{n-1}$ .

e)  $\sum_{n=0}^{\infty} 2^{-n} 3^n$ .

b)  $\sum_{n=1}^{\infty} \frac{2^n}{3^{n-1}}$ .

f)  $\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$ .

c)  $\sum_{n=1}^{\infty} \left(3\left(-\frac{4}{5}\right)^{n-1} - \left(\frac{1}{4}\right)^{n-1}\right)$ .

g)  $\sum_{n=1}^{\infty} \frac{1}{n^2 + 5n + 6}$ .

d)  $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$ .

h)  $\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$ .

4. Write the following decimals as a ratio of two integers.

a)  $0.52525252\dots$

b)  $0.36717171\dots$