Rational Exponents

1. If $a \geq 0$ then $a^{\frac{1}{n}}$ is a real number. Also if $a < 0$ and n is odd then $a^{\frac{1}{n}}$ is a real number.

2. Yes, $16^{\frac{2}{4}} = 16^{\frac{1}{2}}$.

3. An expression with a mixed-number exponent can be simplified by first changing the mixed-number exponent into an improper fraction and then applying the rules for rational exponents. For example, what is $8^{1\frac{1}{3}} = 8^{\frac{4}{3}} = \left(\sqrt[3]{8}\right)^4 = 2^4 = 16$

4. If either a or b or both is equal to 0 then $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$

5. (a) $a^{4/3} - b^{4/3}$ (b) $3|x|\sqrt{|y|}$

6. (a) -4 (b) $\sqrt[5]{3xy^4}$

7. (a) $(13x)^{1/5}$ (b) $(11xy)^{3/2}$

8. (a) $\dfrac{1}{243}$ (b) $\dfrac{1}{\sqrt[7]{256}(xy)^{4/7}}$

9. (a) $x^{2/7}$ (b) $\dfrac{1}{\sqrt[8]{y}}$ (c) $x^7 y^7$ (d) $\sqrt{|xy^3|}$ (e) $\sqrt[12]{x}$

10. An expression with rational exponents is simplified when no parentheses appear, no powers are raised to powers, each base occurs once, and no negative or zero exponents appear.

Multiplying and Simplifying Radical Expressions

1. (a) $29\sqrt{2}\,x$ (b) $13\sqrt[3]{2}$

2. (a) $\sqrt{x^2 - 36}$ (b) $\sqrt{x}$ (c) $2\sqrt{6}|x^5|\sqrt{x}$ (d) $2xy\sqrt[5]{2y^2 z}$

3. (a) $2\sqrt{7}$ (b) $-2y\sqrt[3]{4x^2}$ (c) $xy^5\sqrt[3]{y^2 z^2}$ (d) $y^3\sqrt[5]{y^3}$

Adding, Subtracting, and Dividing Radical Expressions

1. (a) $-6x\sqrt[4]{2x}$ (b) $29x\sqrt{2}$

2. The streets are $\sqrt{15} \approx 3.87$ feet wide.

4. (a) $5\sqrt[3]{7}$ (b) $2\sqrt{17} - \sqrt{19}$ (c) $7y\sqrt[3]{2x}$ (d) $3\sqrt{x-3}$

5. (a) $\dfrac{x\sqrt[3]{x^2}}{5y}$ (b) $\dfrac{2x^2\sqrt[5]{2x^4}}{y^3}$

6. (a) 3 (b) $5x^2\sqrt{2}$ (c) $\dfrac{5\sqrt{xy}}{2}$

7. The perimeter of the rectangle is $24\sqrt{5}$ feet. and the area of the rectangle is 160 square feet.

Multiplying and Rationalizing Denominator
1. $\mathbf{12 + 4\sqrt{2}}$

2. $2^{4/5} = \sqrt[5]{2^4} = \sqrt[5]{16}$

3. $24 - 6\sqrt{3}$ 4. $25 - 2\sqrt{5}$ 5. 30

4. $\dfrac{5\sqrt[3]{2x}}{x}$ 7. $-\dfrac{5a\sqrt{6ab}}{b^3}$ 8. $3\sqrt{6} - 3$ 9. $2 + \dfrac{3}{2}\sqrt{6}$ 10. $\dfrac{y + 6\sqrt{xy + 9x}}{y - 9x}$