## ALGEBRA PROBLEM SESSION \#12 SOLUTIONS

## The Square Root Property and Completing the Square

1. $a \neq 0$ in the quadratic equation $a x^{2}+b x+c=0$, because if $a=0$ then the equation is transformed into a linear equation $b x+c=0$
2. David is incorrect, he must follow the order of operations, which states that Parentheses come first, followed by Exponent, then Multiplication and Division (from left to right) and finally Addition and Subtraction (from left to right), remember the acronym PEMDAS.
3. $x=-1-\frac{\sqrt{3}}{2}$ or $x=1-\frac{\sqrt{3}}{2}$
4. $3 / 4$ needs to be added to $x^{2}+\sqrt{3} x$ to make it a perfect-square trinomial.
5. You can factor the common factor of 4 from the two terms, and then add 1 inside the parentheses on the left, to make the left hand side 4 times a perfect square trinomial and add 4 on the right.

$$
\begin{aligned}
& 4 x^{2}+4 x=-3 \\
& 4\left(x^{2}+x+1\right)=-3+4 \\
& 4\left(x+\frac{1}{2}\right)^{2}=1 \\
& 4 x^{2}+20 x=-5 \\
& 4\left(x^{2}+5 x+\frac{25}{4}\right)=-5+25 \\
& 4\left(x+\frac{5}{2}\right)^{2}=20
\end{aligned}
$$

6. (a) $-\sqrt{\frac{5}{3}}+0 i$ or $\sqrt{\frac{5}{3}}+0 i$
(b) $\quad x=-11$ or $x=5$
(c) $\quad x=-2+2 \sqrt{3}$ or $x=-2-2 \sqrt{3}$
7. (a) 4 needs to be added; $x^{2}+4 x+4=(x+2)^{2}$
(b) 25 needs to be added;. $x^{2}-10 x+25=(x-5)^{2}$
(c) $\frac{16}{100}$ needs to be added; $x^{2}+\frac{4}{5} x+\frac{16}{100}=\left(x+\frac{4}{10}\right)^{2}$
8. 

(a) $x=-7$ or $x=1$
(b) $\quad x=-4-\sqrt{21}$ or $x=-4+\sqrt{21}$
(c) $\quad x=\frac{1}{3}-\frac{2}{3} i$ or $x=\frac{1}{3}+\frac{2}{3} i$

## The Quadratic Formula

1. Ray is incorrect. You need to transform the equation first to get: $2 x^{2}-7 x+3=0$ so that $a=2, b=-7$ and $c=3$.
2. 

(a) $x=-\frac{5}{2} \pm \sqrt{15} i$
(b) $x=-1 \pm \frac{\sqrt{6}}{2} i$
(c) $x=-3 \pm 2 i$
3. (a) $D=b^{2}-4 a c=33 ; 2$ different real solutions
(b) $D=-8 ; 2$ complex conjugate solutions
(c) $D=-80 ; 2$ complex conjugate solutions (purely imaginary)
4. (a) $x=-\frac{1}{2}$ or $x=1$
(c) $x=\frac{1}{2}(3-\sqrt{11})$ or $x=\frac{1}{2}(3+\sqrt{11})$
(b) $\quad x=\frac{1}{4}(-11-\sqrt{33})$ or $x=\frac{1}{4}(-11+\sqrt{33})$
(d) $\quad x=\frac{1}{2}(5-\sqrt{73})$ or $x=\frac{1}{2}(5+\sqrt{73})$
5.
(a) $x^{2}-4 x-12=0$
(b) $x^{2}+64=0$
(c) $x^{2}-45=0$

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6. The discriminant of a quadratic equation $a x^{2}+b x+c=0$ is $D=b^{2}-4 a c$. If $D<0$, then there are 2 different complex conjugate solutions.
If $D=0$, then there is 1 repeated real solution with multiplicity of 2 (double root).
If $D>0$, then there 2 different real solutions. (See Table 8.2)
7. If you are given a quadratic equation, follow the following rules to determine which method to use to solve it: If $a x^{2}+b x+c$ can be factored easily, then factor and use the zero-product principle.
If $a x^{2}+c=0$ and there is no $x$-term, then solve for $x^{2}$ and apply the square root property.
If $u^{2}=d$ and $u$ is a first-degree polynomial, then use the square root property.
If $a x^{2}+b x+c$ cannot be factored or the factoring is too difficult, the use the quadratic formula. (See Table 8.3)
8. Explain how to write a quadratic equation from its solution set. Give an example with your explanation.

If A and B are solutions to a quadratic equation, then the quadratic equation can be obtained by the following equation: $(x-A)(x-B)=0$. For example: $S=\{-5 i, 5 i\}$ then $(x-(-5 i))(x+5 i)=0$, thus $x^{2}+25=0$.

## Quadratic Equations and Functions

## 1.

1. (a)
$(h, k)=(-4,-8)$
(b) $\quad(h, k)=(2,7)$


(d)


2. The function has a minimum value of -11 at $(2,-11)$, a domain of $(-\infty, \infty)$ and range of $[-\mathbf{1 1}, \infty)$.
3. Typo: $s(t)=-16 t^{2}+64 t+200$
(a) After 2 seconds the ball reach its maximum height of 264 feet.
(b) After 6.1 seconds the ball finally hits the ground.
(c) $\boldsymbol{s}(\mathbf{0})=\mathbf{2 0 0}$, the person released the ball 20 feet below the top of the building.
4. The pair of numbers are 8 and 8 and the maximum product is 64 .
5. The length is 100 feet and the width is 50 feet. The largest area that can be enclosed is 5,000 square feet.
6. If the equation of the parabola is given by $\boldsymbol{f}(\boldsymbol{x})=\boldsymbol{a} \boldsymbol{x}^{2}+\boldsymbol{b} \boldsymbol{x}+\boldsymbol{c}$, then the parabola will open upward if a $>0$ and will open downward if $a<0$.
7. A parabola's vertex, $(\boldsymbol{h}, \boldsymbol{k})$, who's equation is in the form $\boldsymbol{f}(\boldsymbol{x})=\boldsymbol{a} \boldsymbol{x}^{2}+\boldsymbol{b} \boldsymbol{x}+\boldsymbol{c}$, then $(\boldsymbol{h}, \boldsymbol{k})$ is given by: $\boldsymbol{h}=-\frac{b}{2 a}$ and $\boldsymbol{k}=\boldsymbol{f}(\boldsymbol{h})=\boldsymbol{f}\left(-\frac{b}{2 a}\right)$. Thus, $f(\boldsymbol{x})=\boldsymbol{x}^{2}-\mathbf{6 x}+8$ has a vertex at $(\boldsymbol{h}, \boldsymbol{k})=(\mathbf{3},-\mathbf{1})$
8. $f(x)=(x+1)^{2}-\mathbf{1}$ is the function related to the graph on the left. $J(x)=-x^{2}-1$ is the function related to the graph on the right.
