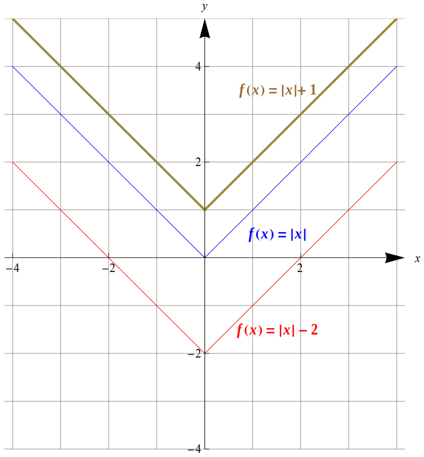
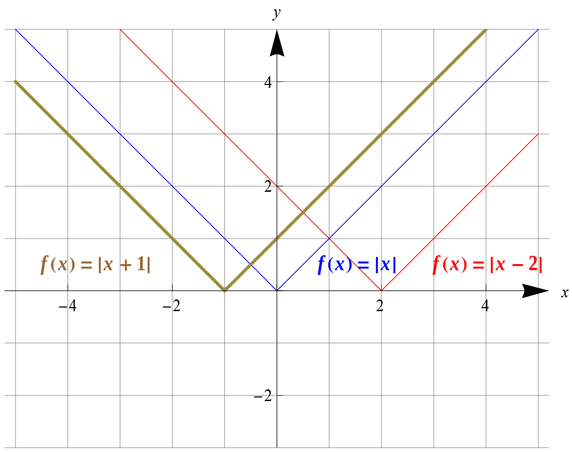
ALGEBRA PROBLEM SESSION #13 SOLUTIONS

Equations that are Quadratic in Form

1. (a) 

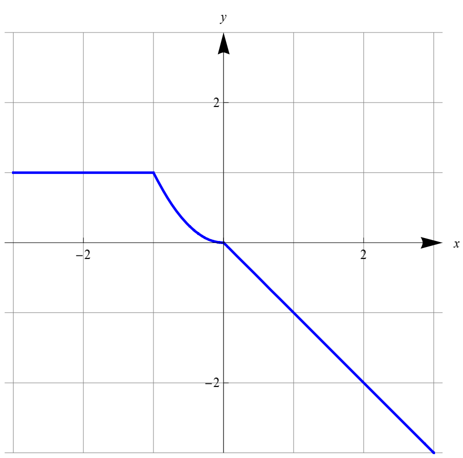
The graph of $y = f(x) + k$ for k, any real number is the graph of $y = f(x)$ vertically shifted k units.
If $k > 0$, then the graph of $f(x)$ is shifted up.
If $k < 0$, then the graph of $f(x)$ is shifted down.

(b) 

The graph of $y = f(x + k)$ for k, any real number is the graph of $y = f(x)$ horizontally shifted k units. If $k > 0$, then the graph of $f(x)$ is shifted left.
If $k < 0$, then the graph of $f(x)$ is shifted right.

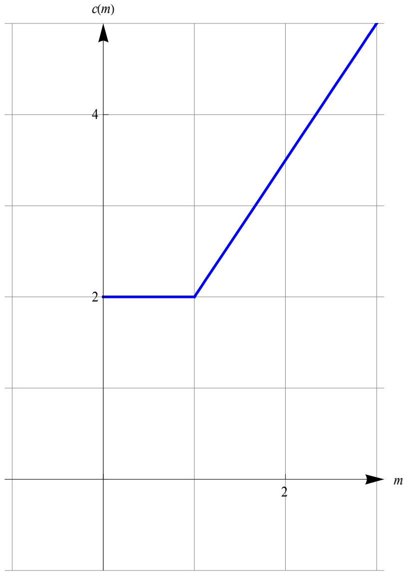
2. a) $y = -|x|$

3. The graph of $f(x) = \sqrt{-x}$ is a reflection of the graph of $f(x) = \sqrt{x}$ about the y-axis.

4. 

The domain of $f(x)$ is $(-\infty, \infty)$.
The range of $f(x)$ is $(-\infty, 1]$.

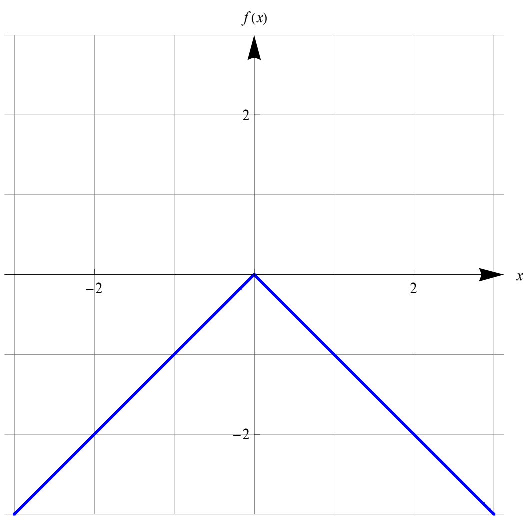
5. $f(x) = \begin{cases} 2x - 1 & , if\ x < 0 \\ 4 & , if\ 0 \leq x < 2 \\ -x + 1 & , if\ x > 2 \end{cases}$

6. 

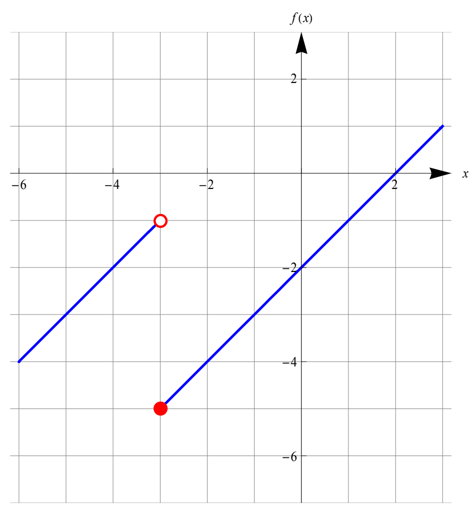
7. $f(x) = \begin{cases} 6 & , if\ x < -1 \\ x+2 & , if\ -1 \leq x < 5 \\ -2x-3 & , if\ x > 5 \end{cases}$

8. a) $f(-3) = -19, f(0) = 3, f(4) = 31$

   b) $g(0) = 5, g(-6) = 1, g(-5) = 0$

9. a) 

Range of the function = $(-\infty, 0]$

b) 

Range of the function = $(-\infty, \infty)$

10. Let the difference quotient, $DQ = \frac{f(x+h)-f(x)}{h}, h \neq 0$ for the given functions.

a) $DQ = 7$ b) $DQ = 2x - 5 + h$ c) $DQ = -6x + 2 - 3h$ d) $DQ = -\frac{1}{2x(h+x)}$

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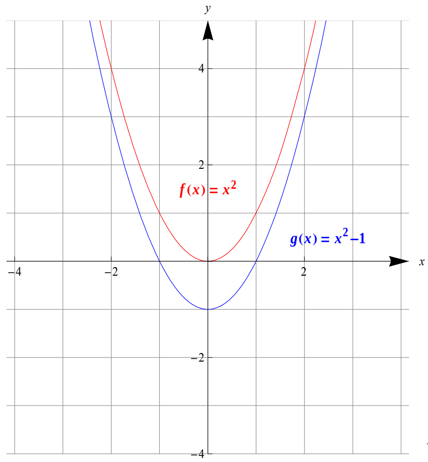
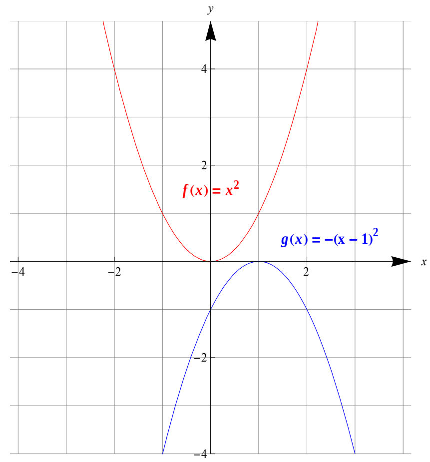
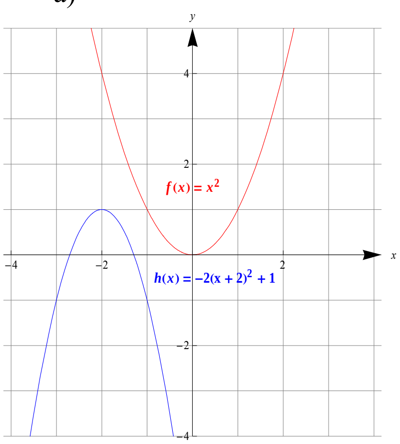
11. a)



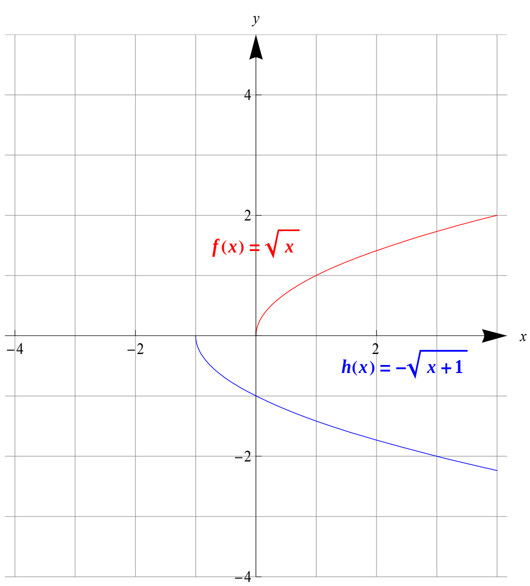
b)



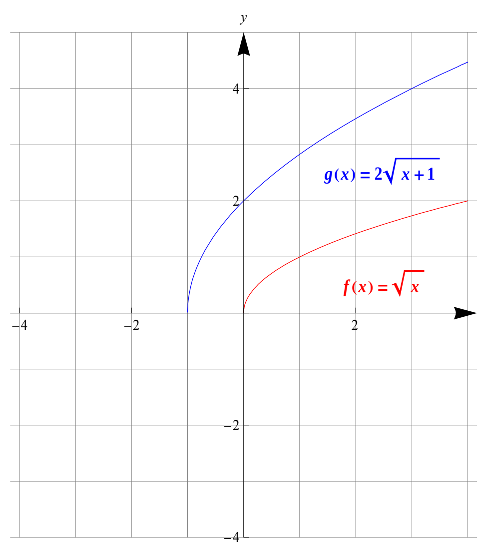
c)

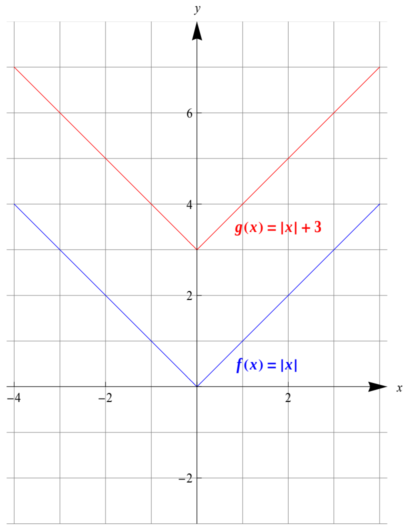


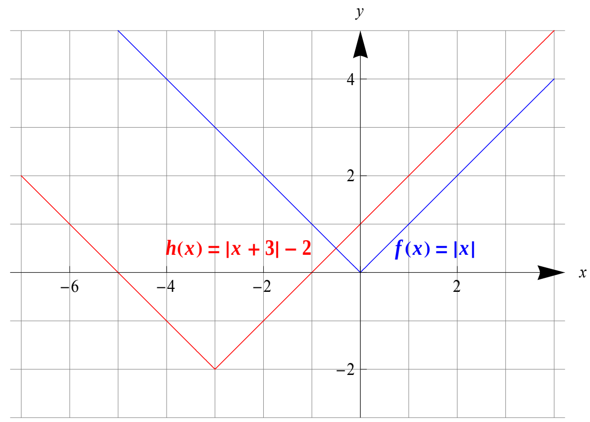
12. (a)

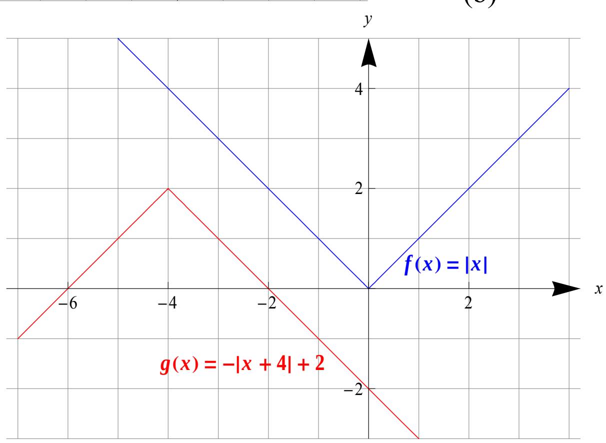


(b)



13. (a) 

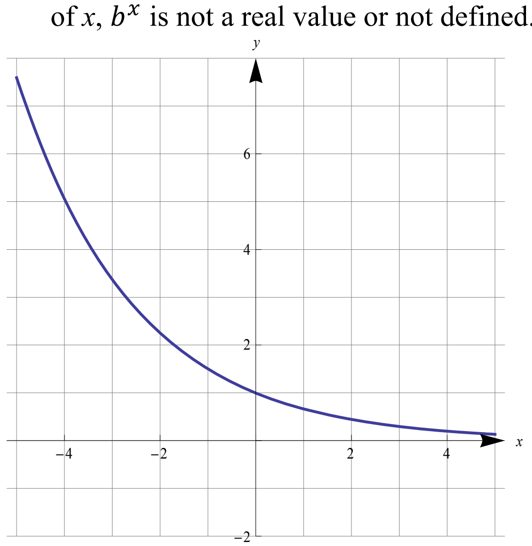
(b) 

(c) 

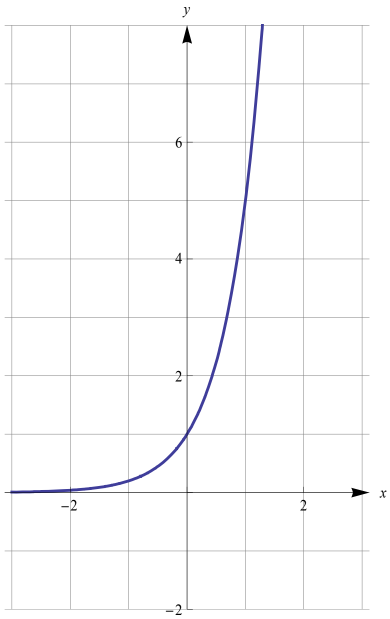
14. (a) $S = \{-\sqrt{5}, -2, 2, \sqrt{5}\}$ (b) $S = \{4, 16\}$ c) $S = \{-\sqrt{5}, 0, \sqrt{5}\}$
(d) $S = \{-5, -4\}$ (e) $S = \{-27, 1\}$

Exponential Functions

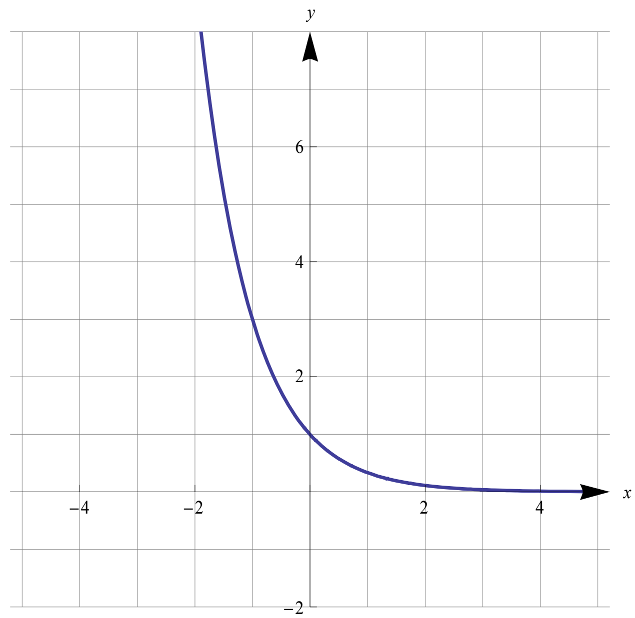
1. (a) In the definition of the exponential function, if b is to equal 0, then $f(x) = a0^x = 0$ a constant function not an exponential function.
(b) In the definition of the exponential function, if b is negative, then the function is not defined since for most values of x, b^x is not a real value or not defined.

2. 

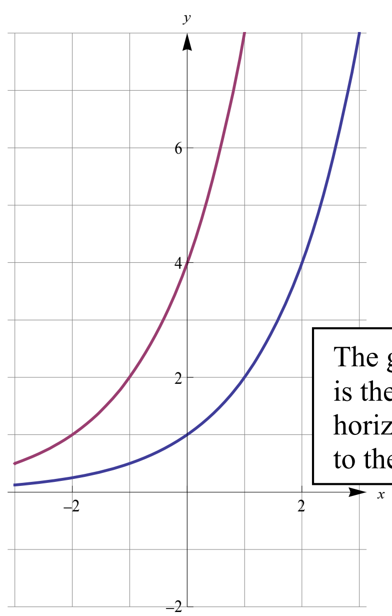
3. (a)



(b)

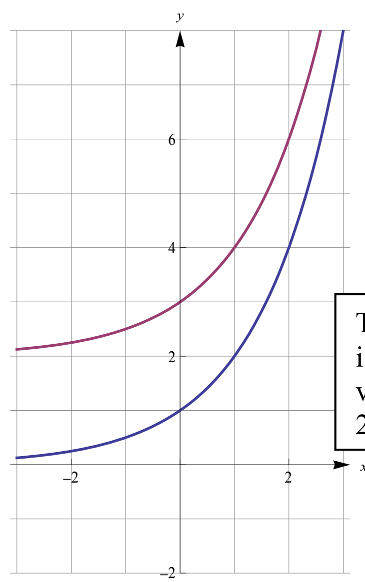


4. (a)



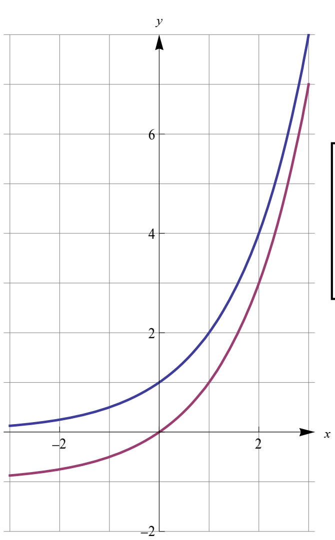
The graph of $g(x)$ is the graph of $f(x)$ horizontally shifted to the left 2 units.

(b)



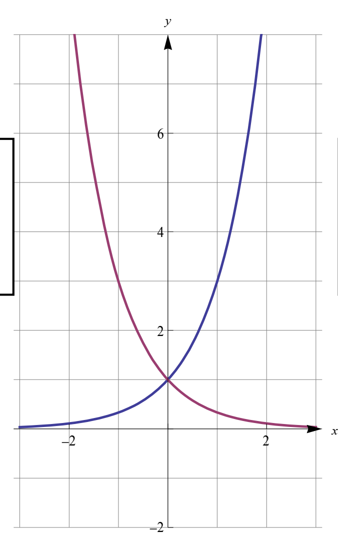
The graph of $g(x)$ is the graph of $f(x)$ vertically shifted up 2 units.

(c)



The graph of $g(x)$ is the graph of $f(x)$ vertically shifted down 2 units.

(d)



The graph of $g(x)$ is the graph of $f(x)$ reflected over the y-axis.

5. Domain = $(-\infty, \infty)$ and Range = $(3, \infty)$.
6. According to the linear model, $f(x) = 0.15x + 1.44$, there were approximately 4.4 million words in the federal tax code in 1975. According to the exponential model, $g(x) = 1.87e^{0.0344x}$, there were approximately 3.7 millions of words in the federal tax code in 1975. The exponential model is a better model in 1975.

7. An exponential function *f* with base *b* is of the form: $f(x) = b^x$ where *b* is a positive constant other than 1 ($b > 0$ and $b \neq 1$) and *x* is any real number.

8. The natural exponential function is an exponential function of the form: $f(x) = e^x$, where $e \approx 2.718281827$ or $e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$

9. Does not make sense, because the number of reductions *x* cannot be any real number, eg. π.

Composite and Inverse Functions

1. The domain of *f*/g is the domain of *f* intersected with the domain of *g*, excluding any points where $g(x) = 0$.

2. The composition of functions is associative.

3. f ∘(g + h) = *f* ∘ g + f ∘ h.

4. (a) 17 (b) 4 (c) 4 (d) 18 (e) –36 (f) $2x^2 - 1$ (g) $4x - 21$

5. (a) $2a^2 - 4a + 3$ (b) $2h^2 - 4h + 3$
   (c) $2(a + h)^2 - 4(a + h) + 3$ (d) $2(a^2 - 2a + h^2 - 2h + 3)$

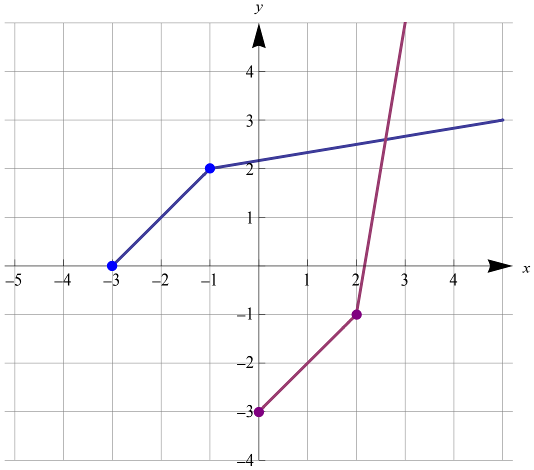
6. $(f - g)(x) = x^2 + 1$ has domain of $(-\infty, \infty)$
   $(f * g)(x) = (x^2 - 4)(2x^2 - 3) = 2x^4 - 11x^2 + 12$ has domain of $(-\infty, \infty)$

7. Typo

8. $2(2x + h) = 4x + 2h$
9. (a) $f\big(g(x)\big) = x$ and $g\big(f(x)\big) = x$ thus $f(x)$ and $g(x)$ are inverses of each other.
   (b) $f\big(g(x)\big) = \frac{3(x+3)}{7} - 7 = \frac{3x}{7} - \frac{40}{7} \neq x$ and $g\big(f(x)\big) = \frac{1}{7}(3x - 4) = \frac{3x}{7} - \frac{4}{7} \neq x$, thus $f(x)$ and $g(x)$ are not inverses of each other.
   (c) $f\big(g(x)\big) = x + 10$ and $g\big(f(x)\big) = x + 10$, thus $f(x)$ and $g(x)$ are not inverses of each other.

10. (a) $f^{-1}(x) = x - 5$ and $f^{-1}(f(x)) = x$ and and $f(f^{-1}(x)) = x$
    (b) $f^{-1}(x) = \sqrt[3]{x + 1}$ and $f^{-1}(f(x)) = x$ and and $f(f^{-1}(x)) = x$
    (c) $f^{-1}(x) = \frac{x+3}{2-x}$ and $f^{-1}(f(x)) = x$ and and $f(f^{-1}(x)) = x$

11. No, this graph does not represent a function that has an inverse function, since it does not pass the horizontal line test (HLT).

12. 

13. Two functions are inverses of each other if $\boldsymbol{f^{-1}(f(x)) = x}$ and and $\boldsymbol{f\left(f^{-1}(x)\right) = x}$.

14. To find the inverse of a one-to-one function follow this procedure:
    1) Change $\boldsymbol{f(x)}$ **to** $\boldsymbol{y}$.
    2) Interchange *y*'s for *x*'s.
    3) Solve for *y*.
    4) Replace $\boldsymbol{f^{-1}(x)}$ **for** $\boldsymbol{y}$.
    5) Check that $\boldsymbol{f^{-1}(f(x)) = x}$ and and $\boldsymbol{f\left(f^{-1}(x)\right) = x}$.

15. The graph of the inverse function of a one-to-one function can be obtained by reflecting each point of the one-to-one function over the graph of the identity function $\boldsymbol{f(x) = x}$ **or** $\boldsymbol{y = x}$.