ALGEBRA PROBLEM SESSION #2 SOLUTIONS

INTRODUCTION TO FUNCTIONS

1. A function is a relation from a set, called the domain, to another set, called the range, such that each element in the domain corresponds to exactly one element in the range.

2. The relation {(2, 3), (2, 4), (2, 5)} is not a function since 2 corresponds to more than one element in the range, ie. 2 is mapped to 3, 4 and 5 in the range.

3. The relation is {(0, 3), (1, 3), (2, 3)} is a function.

4. No, $-12 = P(2) + P(3) \neq P(2 + 3) = 0$

5. No, $P(-1) = -2$

6. a. $f(x) + 2 = 4x^2 + x - 5$, 2 is being added to $f(x)$
   b. $f(x + 2) = 11 + 17x + 4x^2$, the function $f(x)$ is being evaluated at $x + 2$
   c. $f(x) + f(2) = 4 + x + 4x^2$,
   the function $f(x)$ is being evaluated at x and at 2 and then these two evaluations are added

7. If $= -\frac{7}{8b}$, then $f(a + b) = f(a) + f(b)$.

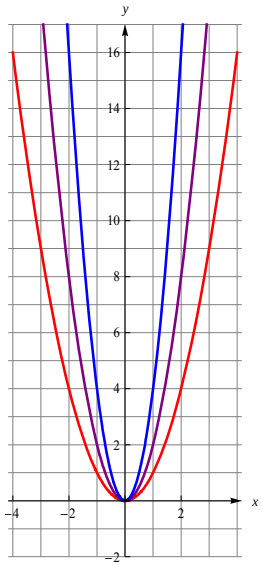
8. $f(6) = 35$

Graphs of Functions

1. The graph of a vertical line $x = a$ does not pass the VLT or there is more than one y value corresponding to the x value of a.

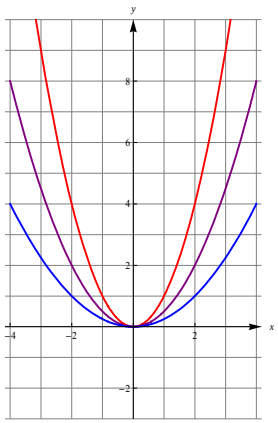
2. The vertical line test for determining if the graph of a relation is a function works because if a vertical line intersects the graph of a relation at more than one point then there would be 2 or more y values for the same x value, thus the relation would not be a function. Otherwise the relation is a function.

3.



The graphs $f(x) = 2x^2$ and $f(x) = 4x^2$ are vertical stretches of the graph of $f(x) = x^2$ by a factor of 2 and 4 respectively.

4.



The graphs $f(x) = \frac{1}{2}x^2$ and $f(x) = \frac{1}{4}x^2$ are vertical shrinkings of the graph of $f(x) = x^2$ by a factor of ½ and ¼ respectively.

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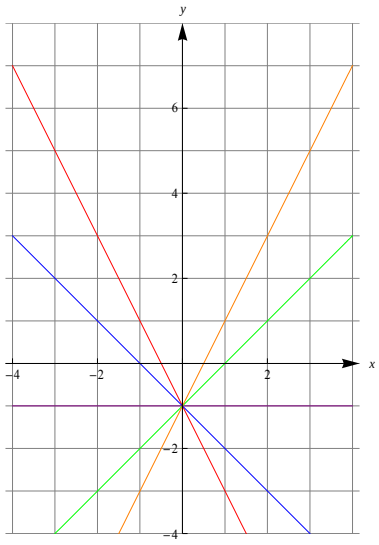
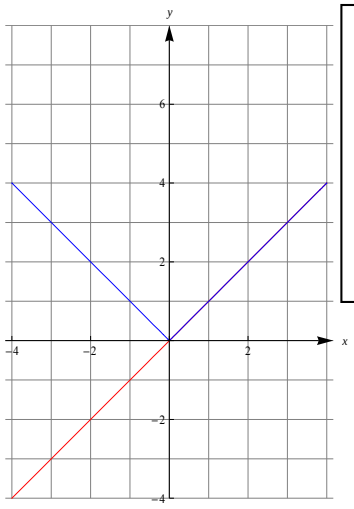
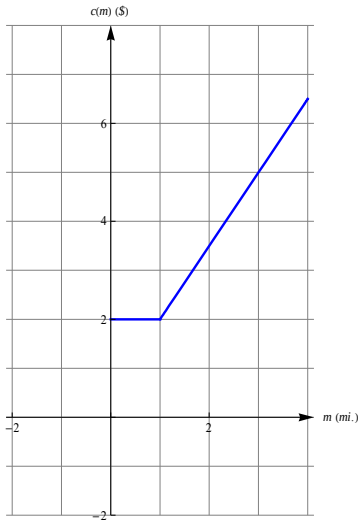
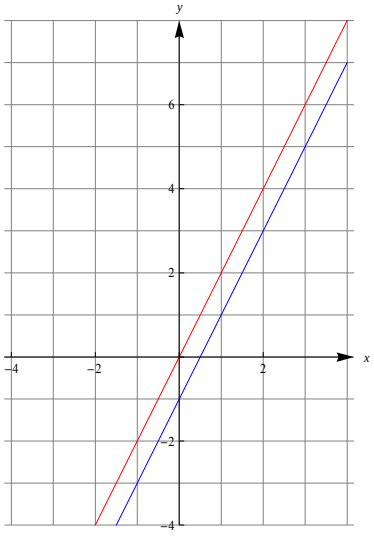
5. 

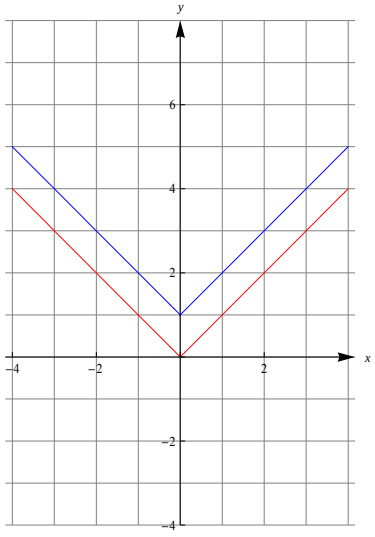
The value of m affects the slope of the line crossing at the point (0, -1).

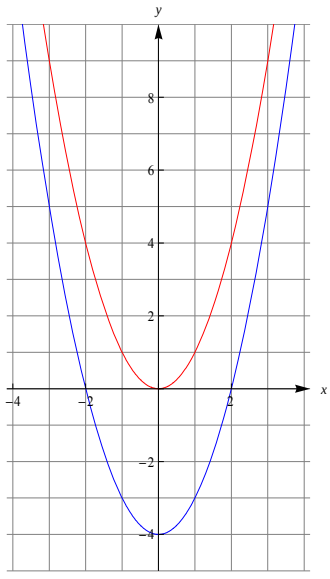
6. 

When $x \geq 0$ the graph of $f(x) = |x|$ and $g(x) = x$ are the same and when $x < 0$ the graph of $f(x) = |x|$ is the graph of $g(x) = x$ reflected over the x-axis.

7. 

8. 

9. 

10. 

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1. The domain of a function is found by finding all the *x* values for which the function is defined for.

2. All reals except the number 5. $\{x \in \mathbb{R} | x \neq 5\}$

3. $\{x \in \mathbb{R} | x \neq -5, x \neq 7\}$

4. $(f+g)(x) = x - 4 \qquad (f+g)(4) = 53$

5. $\{x \in \mathbb{R} | x \neq 6\}$

6. a. $\{x \in \mathbb{R}\}$ b. $\{x \in \mathbb{R} | x \neq 2\}$

7. a. $f(-4) + g(-5) = 16$ b. $(f-g)(x) = x^2 + 3x + 1$ and $(f-g)(4) = 25$
   c. $(fg)(-3) = 18$ d. $(f/g)(-2) = 0$

8. $f(x) + g(x) = x = g(x) + f(x)$ and $f(x) - g(x) = 3x + 2 \neq g(x) - f(x) = -3x - 2$

9. For the indicated functions f and g, find the functions f + g, f – g, fg, and f/g and find their domains.

a. $f(x) = x + \frac{1}{x}$, $g(x) = x - \frac{1}{x}$

	Domain
$(f+g)(x) = 2x$	$\{x \in \mathbb{R} \mid x \neq 0\}$
$(f-g)(x) = \frac{2}{x}$	$\{x \in \mathbb{R} \mid x \neq 0\}$
$(fg)(x) = (x - \frac{1}{x})(x + \frac{1}{x})$	$\{x \in \mathbb{R} \mid x \neq 0\}$
$\left(\frac{f}{g}\right)(x) = \frac{x+\frac{1}{x}}{x-\frac{1}{x}}$	$\{x \in \mathbb{R} \mid x \neq 0, 1, -1\}$

b. $f(x) = x - 1 \qquad g(x) = x - \frac{6}{x-1}$

	Domain
$(f+g)(x) = 2x - \frac{6}{x-1} - 1 = \frac{2x^2-3x-5}{x-1}$	$\{x \in \mathbb{R} \mid x \neq 1\}$
$(f-g)(x) = \frac{6}{x-1} - 1 = \frac{7-x}{x-1}$	$\{x \in \mathbb{R} \mid x \neq 1\}$
$(fg)(x) = (x-1)(x - \frac{6}{x-1}) = x^2 - x - 6$	$\{x \in \mathbb{R} \mid x \neq 1\}$
$\left(\frac{f}{g}\right)(x) = \frac{x-1}{x-\frac{6}{x-1}} = \frac{x^2-x-6}{x-1}$	$\{x \in \mathbb{R} \mid x \neq 1\}$

c. $f(x) = 1 - \frac{x}{|x|} \qquad g(x) = 1 + \frac{x}{|x|}$

	Domain
$(f+g)(x) = 2$	$\{x \in \mathbb{R} \mid x \neq 0\}$
$(f-g)(x) = -\frac{2x}{\|x\|}$	$\{x \in \mathbb{R} \mid x \neq 0\}$
$(fg)(x) = \frac{(\|x\|-x)(\|x\|+x)}{\|x\|^2}$	$\{x \in \mathbb{R} \mid x \neq 0\}$
$\left(\frac{f}{g}\right)(x) = \frac{\|x\| - x}{\|x\| + x}$	$\{x \in \mathbb{R} \mid x \neq 0, x \neq -1\}$